Coded Multiband OFDM with Tone Interference

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I. Introduction

We consider the convolutionally-coded Multiband Orthogonal Frequency Division Multiplexing (MB-OFDM) system for Ultra Wideband (UWB) communications [1], which operates over quasi-static, frequency-selective wireless channels [2]. Classical bit error rate (BER) analysis techniques for coded systems [3] are not applicable in this setting because the channel is (a) non-ideally interleaved (resulting in non-zero correlation between adjacent coded bits), and (b) quasi-static (which limits the number of distinct channel gains to the number of OFDM subcarriers).

We present a method for determining the average BER of this system, and note that it is applicable to the general class of coded OFDM systems operating over frequency-selective quasi-static channels. Furthermore, in our analysis we account for narrowband interference (modeled as a sum of tone interferers, a reasonable model for evaluating the effect of one or more interferers with narrow bandwidth as compared to the OFDM subcarrier spacing). The effects of such narrowband interference are of particular practical interest due to the frequency reuse and resultant interference inherent to UWB-based communication schemes.

II. Problem Formulation and Solution

System Model: MB-OFDM employs 128 subcarriers and hops over 3 sub-bands, forming an equivalent 384 subcarrier OFDM system (with $N = 300$ data-carrying subcarriers). Channel coding consists of standard convolutional codes and interleaving. Interleaved coded bits are mapped to quaternary phase-shift keying (QPSK) symbols using Gray labeling. We use R_c to denote the effective code rate after puncturing [1].

The modulated symbols $\boldsymbol{x}$ are transmitted through a quasi-static fading channel with frequency-domain channel gains $\boldsymbol{h} = [h_1\ h_2\ \ldots\ h_N]$ and correlation matrix $\boldsymbol{\Sigma_{hh}}$. Writing $\boldsymbol{H} = \mathrm{diag}(\boldsymbol{h})$, where $\mathrm{diag}(\boldsymbol{h})$ denotes a matrix with the elements of $\boldsymbol{h}$ on the main diagonal, we can express the received symbols $\boldsymbol{r}$ (after the DFT) as $\boldsymbol{r} = \boldsymbol{Hx}+\boldsymbol{J}+\boldsymbol{n}$, where $\boldsymbol{J}$ is the interference and $\boldsymbol{n}$ is a vector of independent complex additive white Gaussian noise (AWGN) variables with variance $\mathcal{N}_0$. For a meaningful performance analysis of the MB-OFDM proposal, we consider the quasi-static channel model CM1 developed under IEEE 802.15 for UWB systems [2]. By ignoring the constant-per-realization "outer" lognormal shadowing of this model, the elements of $\boldsymbol{h}$ are well-approximated as zero-mean complex Gaussian random variables [4]. We assume perfect timing and frequency synchronization. The receiver employs a soft-output detector followed by a deinterleaver, a depuncturer, and standard Viterbi decoding.

Narrowband interference is modeled as the sum of N_i tone interferers $i(t) = \sum_{k=1}^{N_i} i_k(t) = \sum_{k=1}^{N_i} g_k e^{j(2\pi f_k t+\phi_k)}$, where the k^{th} tone interferer has amplitude g_k, frequency f_k, and initial phase ϕ_k. Throughout this paper we will assume that $g_k = 1$, i.e., that the interferers are transmitted through a constant-amplitude channel to the receiver. After sampling $i(t)$ with the OFDM system sampling rate $1/T$, we obtain the frequency-domain equivalent $\boldsymbol{J}$ of the interfering signal, given by $\boldsymbol{J} = \mathrm{DFT}([i(0)\quad i(T)\quad i(2T)\quad \ldots\quad i((N-1)T)]^T)$, where $\mathrm{DFT}(\cdot)$ denotes the Discrete Fourier Transform and $[\cdot]^T$ denotes vector transposition.

Error Vectors: Let $\mathcal{E}$ be the set of all L vectors $\boldsymbol{e}_\ell$ $(1 \le \ell \le L)$ of code output (after puncturing) associated with input sequences with Hamming weight less than $w_{\max}$, i.e., $\mathcal{E} = \{\boldsymbol{e}_1, \boldsymbol{e}_2, \ldots, \boldsymbol{e}_L\}$. Let l_ℓ and a_ℓ be the length of $\boldsymbol{e}_\ell$ and the number of information bit errors associated with $\boldsymbol{e}_\ell$, respectively. We term $\boldsymbol{e}_\ell$ an "error vector" and $\mathcal{E}$ the set of error vectors. Note that $\mathcal{E}$ can be straightforwardly obtained from the transfer function of the code.

Pairwise Error Probability (PEP): Consider error events starting in a given position i of the codeword $(1 \le i \le L_c)$. For each error vector $\boldsymbol{e}_\ell$ $(1 \le \ell \le L)$, form the full error codeword $\boldsymbol{q}_{i,\ell} = [0\ 0\ \ldots\ 0\ \boldsymbol{e}_\ell\ 0\ 0\ \ldots\ 0]^T$ of length L_c by padding $\boldsymbol{e}_\ell$ with $(i-1)$ and $(L_c - l_\ell - i + 1)$ zeros on the left and right, respectively. Given the error codeword $\boldsymbol{q}_{i,\ell}$ and given that codeword $\boldsymbol{c}$ is transmitted, the competing codeword is given by $\boldsymbol{v}_{i,\ell} = \boldsymbol{c} \oplus \boldsymbol{q}_{i,\ell}$ (where $\oplus$ denotes XOR). Let $\boldsymbol{z}_{i,\ell}$ be the vector of QPSK symbols associated with $\boldsymbol{v}^\pi_{i,\ell}$ (the interleaved version of $\boldsymbol{v}_{i,\ell}$). Noting that only the $\eta_{i,\ell}$ non-zero terms of $(\boldsymbol{x} - \boldsymbol{z}_{i,\ell})$ contribute to the PEP (and suppressing the dependence of η on i and ℓ for notational clarity), we let $\boldsymbol{x'}$, $\boldsymbol{z'}_{i,\ell}$, and $\boldsymbol{J'}$ represent the transmitted symbols, received symbols, and interferences corresponding to the η non-zero distances of $(\boldsymbol{x} - \boldsymbol{z}_{i,\ell})$, respectively, and form $\boldsymbol{\Sigma_{h'h'}}$ by extracting the relevant portions of $\boldsymbol{\Sigma_{hh}}$. Let $\boldsymbol{D} = \mathrm{diag}(\boldsymbol{x'} - \boldsymbol{z'}_{i,\ell})$ be the diagonal matrix of non-zero distances, and let $\boldsymbol{R_{h'h'}} = \boldsymbol{D\Sigma_{h'h'}D}^H$. Using Laplace transform techniques [5], it is possible to show that the PEP for the ℓth error vector starting in the ith position is given by

$$\mathrm{PEP}_{i,\ell} = \frac{1}{2\pi j}\int_{c-j\infty}^{c+j\infty} \frac{\exp[-s\boldsymbol{\mu}^H(\boldsymbol{A}^{-1}+s\boldsymbol{R})^{-1}\boldsymbol{\mu}]}{\det(\boldsymbol{I}_{2\eta}+s\boldsymbol{RA})}\frac{\mathrm{d}s}{s} \qquad (1)$$

with

$$\boldsymbol{\mu} = \begin{bmatrix}\boldsymbol{0}_{\eta\times 1}\\ \boldsymbol{J'}\end{bmatrix},\ \boldsymbol{A} = \begin{bmatrix}\boldsymbol{I}_\eta & -\boldsymbol{I}_\eta\\ -\boldsymbol{I}_\eta & \boldsymbol{0}_\eta\end{bmatrix},\ \boldsymbol{R} = \begin{bmatrix}\boldsymbol{R_{h'h'}} & \boldsymbol{0}_\eta\\ \boldsymbol{0}_\eta & \mathcal{N}_0\boldsymbol{I}_\eta\end{bmatrix}\ ,$$

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where $\boldsymbol{I}_\eta$ and $\mathbf{0}_\eta$ denote the identity matrix and the all-zero matrix of dimension $\eta \times \eta$, respectively, $\mathbf{0}_{\eta\times 1}$ denotes the all-zero column vector of length η, $\det(\cdot)$ denotes the determinant of a matrix, and $(\cdot)^H$ denotes the Hermitian transpose. We note that (1) may be solved efficiently via numerical integration using a Gauss-Chebyshev quadrature rule [5].

Average BER Approximation: As discussed in detail in [6], the average BER performance of coded MB-OFDM can be well approximated as

$$\bar{P} = \frac{1}{L_c}\sum_{i=1}^{L_c}\sum_{\ell=1}^{L} a_\ell \mathrm{PEP}_{i,\ell}\ . \qquad (2)$$

III. Numerical Results and Discussion

We present results for the MB-OFDM system operating at 320 Mbps ($R_c = 1/2$ after puncturing) over the CM1 channel. As mentioned above and in [4], [6], the "outer" lognormal shadowing can be ignored during the analysis, and reintroduced via numerical integration of (2) over the lognormal distribution. We concentrate on the case of $N_i = 1$ interferers, in order to examine the effect of interferer frequency f_1 and the signal-to-interference ratio $\mathrm{SIR} = \mathbb{E}\left(||\boldsymbol{Hx}||^2\right)/\mathbb{E}\left(||\boldsymbol{J}||^2\right)$, where $\mathbb{E}(\cdot)$ denotes expectation.[1] Without loss of generality we place f_1 between the 52^{nd} and 53^{rd} MB-OFDM subcarriers.

Figure 1 shows the BER versus $10\log_{10}(\bar{E}_b/\mathcal{N}_0)$ (the average signal-to-noise ratio per information bit), and clearly illustrates the performance degradation associated with decreasing SIR. Interestingly, the BER when f_1 is exactly the frequency of a subcarrier is worse than when f_1 lies exactly between two subcarriers. For comparison, the no-interference ($\mathrm{SIR} = \infty$) performance is obtained from two different methods: that of Section II (thick solid line), and an alternative per-realization based method [6] (thick dash-dotted line). We note that these two methods are in close agreement. Furthermore, a good match with simulation-based results (not shown here) has been observed.

In Figure 2 we fix the $10\log_{10}(\bar{E}_b/\mathcal{N}_0) = 17$ dB and focus on the effect of varying f_1. As discussed in the previous paragraph and seen in this figure, the best-case performance is obtained when f_1 lies exactly between two OFDM subcarriers, while the performance degrades as f_1 approaches a subcarrier frequency.

In conclusion, the method presented herein provides an efficient means of evaluating the average performance of coded multicarrier systems. This technique constitutes an alternative to simulation-based approaches which, for quasi-static channels, require simulation of large numbers of channel realizations, and are thus very resource-intensive.

[1] Note that the SIR according to this definition is an average over all the subcarriers, so the SIR for a specific subcarrier may be much higher/lower than the average. For example, in the 384-subcarrier MB-OFDM system with one interferer directly on a subcarrier, the SIR of the affected subcarrier will be ≈ 26 dB lower than the average SIR (since the interference on all other subcarriers is zero).

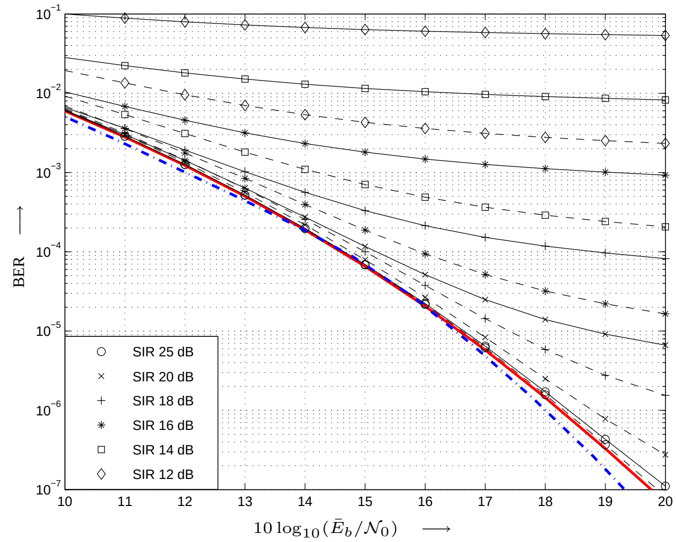

Fig. 1. BER vs. $10\log_{10}(\bar{E}_b/\mathcal{N}_0)$ for varying SIR. Interferer positions 52.0 (solid lines) and 52.5 (dashed lines). For comparison: $\mathrm{SIR} = \infty$ from average method (thick solid line), and realization-based method averaged over 10,000 channels [6] (thick dash-dotted line). UWB channel CM1 with lognormal shadowing, $R_c = 1/2$, QPSK.

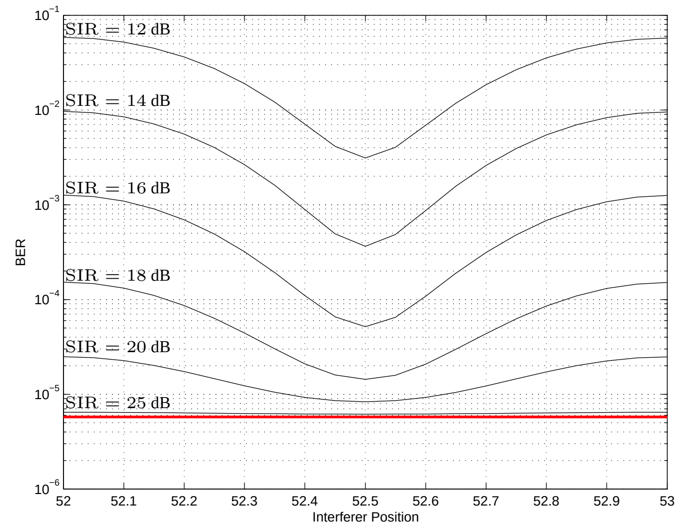

Fig. 2. BER vs. interferer position for $10\log_{10}(\bar{E}_b/\mathcal{N}_0) = 17$ dB. For comparison: $\mathrm{SIR} = \infty$ (thick solid line). UWB channel CM1 with lognormal shadowing, $R_c = 1/2$, QPSK.

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