

# Performance Analysis of Multiband OFDM for UWB Communication



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# Organization of the Talk

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- Overview of Ultra Wideband (UWB)
- Multiband OFDM proposed standard
- UWB channel models
- Channel Estimation for Multiband OFDM
- Performance Measures
  - Information Theory: capacity and cutoff rate
  - Simulation: bit error rates
- Results
- Conclusions

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## Overview of UWB

## Precise Definition of UWB (FCC)

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“an intentional radiator that, at any point in time, has a fractional bandwidth equal to or greater than 0.20 or has a **bandwidth equal to or greater than 500 MHz**, regardless of the fractional bandwidth.”

# What is Ultra Wideband Radio

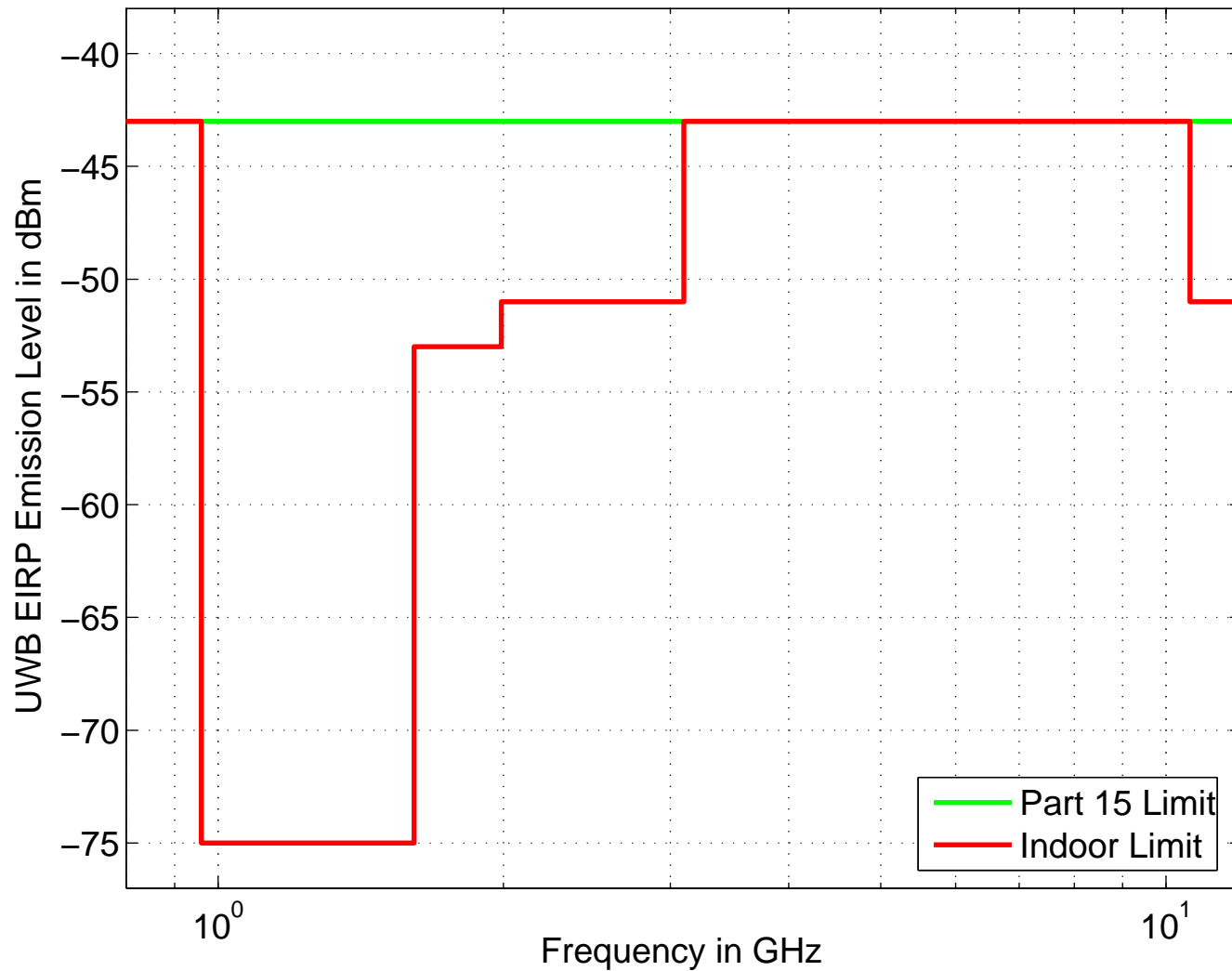
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- Use a very large bandwidth (at least 500 MHz)
- Transmit at very low power
- This allows for operation over currently allocated spectrum

## Applications:

- low power, short range, high speed wireless
- wireless desktop
- Personal Area Networks
- also has self-locationing and radar applications

# UWB Spectral Mask



Adapted from G.R. Aiello and G.D. Rogerson “Ultra-Wideband Wireless Systems”,  
*IEEE Microwave Mag.*, June 2003

The main implementations that have been considered by the IEEE for high-rate PAN standard:

- ~~Impulse radio~~ (first, most research done)
- DS-CDMA based approach (“DS-UWB”)
- OFDM (“Multiband OFDM”)

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## Multiband OFDM proposed standard

# OFDM Refresher

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- OFDM = “Orthogonal Frequency Division Multiplexing”
- Alternative to single-carrier modulation for high data rates
- Use  $N$  narrowband carriers, each at a fraction of total data rate
- Implementation: IFFT/FFT to combine/seperate carriers
- Cyclic prefix prevents inter OFDM-symbol interference, result is  $N$  non-interfering channels (in frequency domain)
- Proven technology — used in (for example):
  - WLAN standards (eg 802.11g, HiperLAN/2)
  - DSL modems (wireline people call it DMT)

Frequency-selective fading channel  $\rightarrow$   $N$  flat fading channels

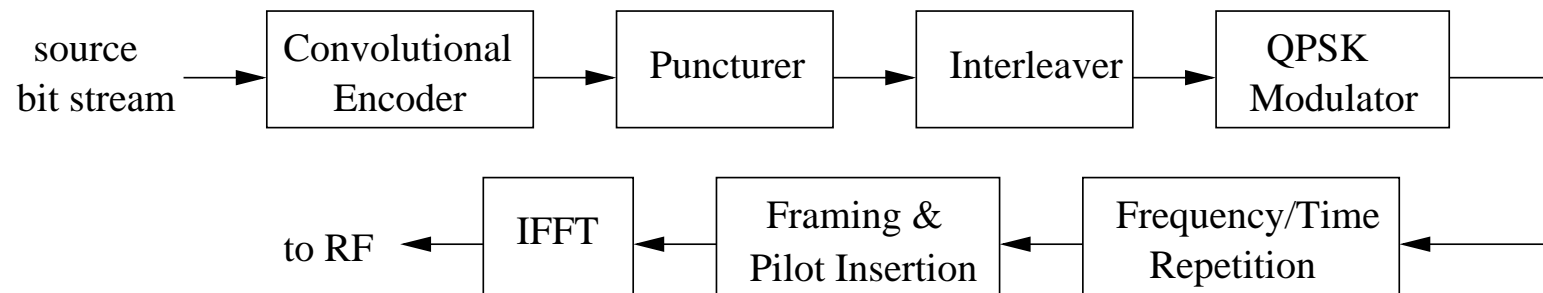
# Multiband OFDM

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- Split the 3.1–10.6 GHz band into 528 MHz subbands
- In the 528 MHz band, use OFDM ( $N = 128$  carriers)
- QPSK modulation on each carrier
- Conv. Codes + interleaving  $\rightarrow$  bit-interleaved coded modulation (BICM)
- 10 different data rates from 53.3 Mbps to 480 Mbps
- Different rates supported via code puncturing and optional repetition in time/frequency
- Frequency hop between bands every OFDM symbol

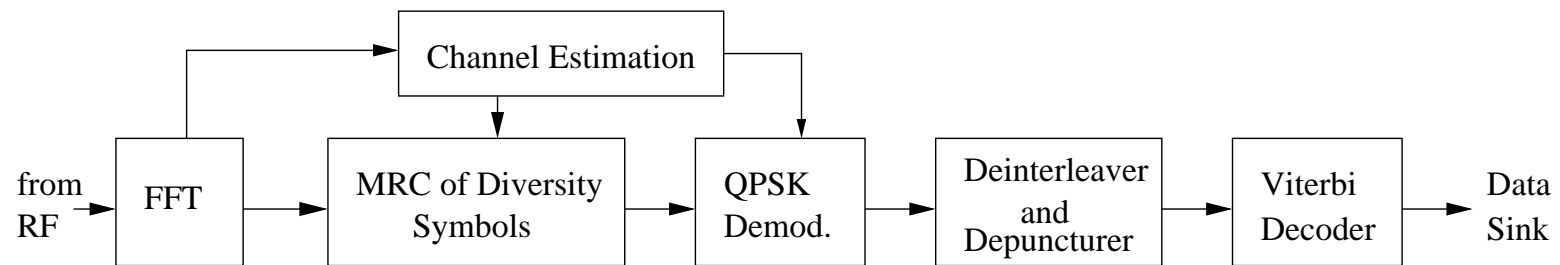
# Multiband OFDM Transmitter

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# Multiband OFDM Receiver

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# Channel Estimation with MB-OFDM

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- Channel Estimation symbols in data packet header
- Pilot tones in every OFDM symbol
- With these, can implement pilot-symbol aided channel estimation (PSAM)
- If channel is slowly time-varying, can do estimation based on the header estimation symbols (more on this later)

Multiband OFDM is based on well-known communication techniques, so we can apply existing analysis tools to this problem

- BICM analysis
- OFDM analysis

**BUT** most existing literature assumes Rayleigh or Rician channels. As we will see, the UWB channel is more complex.

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## UWB channel models

# Channel Model Development

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- IEEE 802.15 (Personal Area Networks) developed channel models for UWB
- Based on existing literature and measurement campaigns for wideband channels
- They chose a model to fit these data
- On UWB channel, multipath rays arrive in clusters

The real-valued RF channel impulse response given by

$$h(t) = X \sum_{l \geq 0} \sum_{k \geq 0} \alpha_{k,l} \delta(t - T_l - \tau_{k,l})$$

with

- Cluster and ray arrival times ( $T_l$  and  $\tau_{k,l}$ ) are conditionally exponentially distributed
- Ray magnitudes ( $\alpha_{k,l}$ ) are exponentially decaying, lognormally distributed
- $X$  a lognormal r.v.

- Impulse response composed of clusters of rays
- Cluster and ray interarrival times exponentially distributed (decay factors chosen to meet some power decay profile)
- Ray amplitudes are lognormal random variables
- Each cluster also undergoes a lognormal fading
- Total impulse response energy is normalized
- Entire impulse response has an “outer” lognormal fading

## Four different channel models

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Four separate sets of parameters available, depending on scenario being considered

- CM1: 0–4 meters, line-of-sight (LOS) channel
- CM2: 0–4 meters, non-LOS channel
- CM3: 4–10 meters, non-LOS channel
- CM4: “extreme non-LOS multipath channel”

# Channel Model Analysis

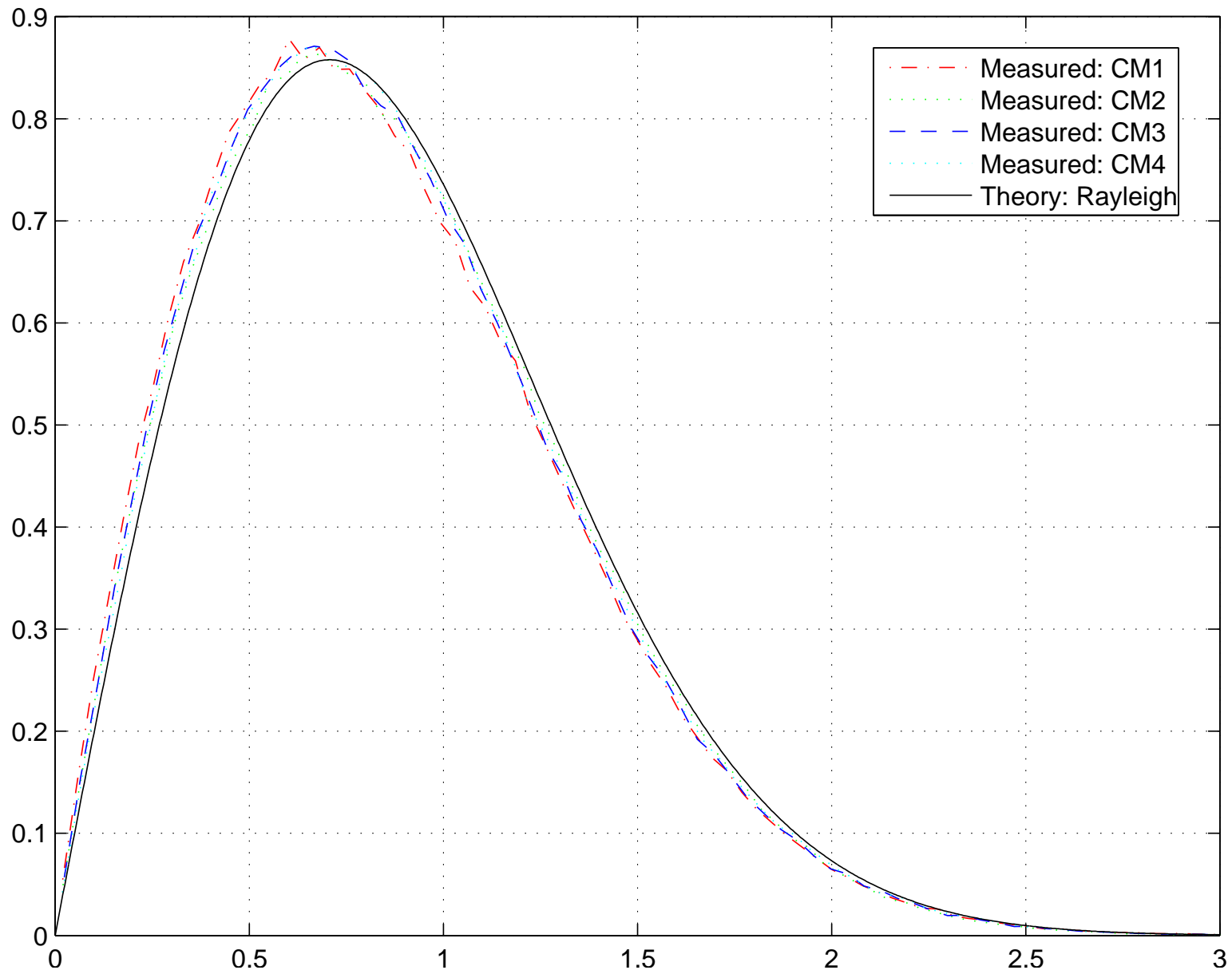
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We will exclusively consider  $H_i$ , the frequency-domain channel responses of each carrier — sample  $h(t)$  at 528Msps and take Fourier transform. We also ignore the “outer” lognormal term  $X$ .

We study

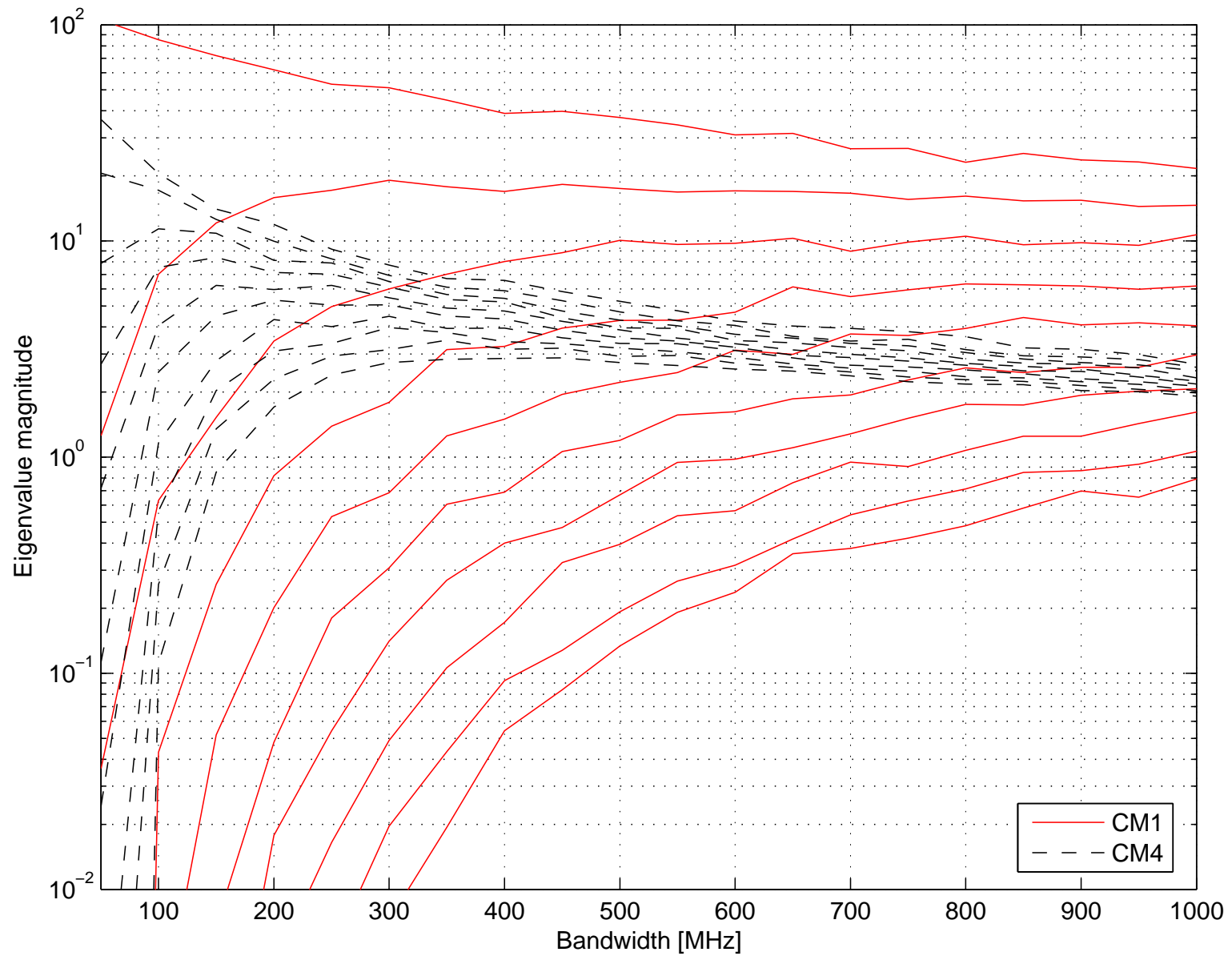
- Marginal distribution of  $|H|$ , i.e.  $p(|H_i|)$
- Subcarrier correlation matrix  $\mathbf{R}_{\mathbf{H}\mathbf{H}}$  (we will examine its eigenvalues here)

# $p(|H_i|)$ Measurements



- $H_i$  are almost Rayleigh distributed! (neglecting  $X$ )
- This is expected —  $H_i$  is a superposition of relatively many mutually-independent time-domain multipath components
- For a particular lognormal fading  $X$ , **H is well approximated by a Rayleigh fading channel**

# First 20 eigs of $R_{HH}$ (every 2nd shown)



Diversity order is (roughly) the number of “useful” (ie not small) eigenvalues

- Diversity order is increasing with bandwidth
- At 500 MHz, have diversity order  $\geq 20$  for CM4,  $\geq 10$  for CM1
- Our code ( $R = 1/3, k = 7, d_{free} = 15$ ) can only exploit at most diversity order 15, so bandwidth higher than 500 MHz not useful except with low-rate time/freq repetition modes
- 528 MHz bandwidth of MB-OFDM a good tradeoff between complexity and achievable diversity order
- CM1 advantageous for high-rate modes (lots of puncturing, therefore cannot exploit a large diversity order) due to larger dominating eigenvalues

## Channel model — Take-home message

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- For a particular lognormal fading  $X$ ,  $\mathbf{H}$  is well approximated by a Rayleigh fading channel
- The 528 MHz bandwidth of MB-OFDM has sufficient diversity to fully exploit the coding gain of the codes we are using

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## Effect of Channel Estimation

# Channel Estimation

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- Effects of imperfect channel state information (CSI)?
- We concentrate on estimating the channel via transmitted pilot symbols. Two possible methods:
  1. Pilot carriers transmitted in each OFDM symbol
  2. Pilot symbols transmitted in data packet header
- Because the UWB channel is slowly time varying, we will use **pilot symbols** in the packet header

## Channel Estimation Implementation

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- Least-squares error (LSE) channel **impulse response** estimator
- Estimating in time-domain allows us to exploit fact that impulse response length  $L \leq N$
- For properly designed OFDM system,  $L$  less than the cyclic prefix length

We skip maths here, just note that performance depends only SNR and

$$\eta = \frac{L}{NP}$$

where

- $L$  impulse response length
- $N$  number of carriers
- $P$  number of pilot OFDM symbols

(for proposed standard,  $L = 32, N = 128, P = 2 \rightarrow \eta = 0.25$ )

## Equivalent SNR with Channel Estimation

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With estimator as described, we can derive the conditional pdf at the receiver

$$p(Y_i|\hat{H}_i, X_i) = \frac{1}{\pi(\sigma_N^2(\eta\mu^2 + 1))} \exp\left(-\frac{|Y_i - X_i\hat{H}_i\mu|^2}{\sigma_N^2(\eta\mu^2 + 1)}\right)$$

where  $\hat{H}_i$  is the channel estimate and  $\mu$  is the correlation between the true channel  $H_i$  and the estimate  $\hat{H}_i$ .

This is a Gaussian density — we can represent the system with estimation operating at  $\gamma_t$  as a system with perfect CSI at some  $\gamma_e$

$$\gamma_e = \frac{\gamma_t}{\eta\left(1 + \frac{1}{\gamma_t}\right) + 1}$$

Note as  $\gamma_t \rightarrow \infty$ , SNR loss due to estimation goes to  $1/(\eta + 1)$

- Estimate slowly time-varying UWB channel using OFDM pilot symbols in the packet header
- LSE estimator, performance depends solely on factor  $\eta$
- We can consider the system with estimation operating at SNR  $\gamma_t$  as a system with perfect CSI at some equivalent SNR  $\gamma_e$

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## Performance Measures

We take two approaches:

1. Information-theoretic analysis
  - Channel capacity (ultimate limits)
  - Cutoff rate (practical limit of convolutional codes)
2. System simulations

For both approaches we consider perfect CSI as well as imperfect CSI due to channel estimation using OFDM pilot symbols

Channel Capacity  $C$ :

- For any transmission rate  $R < C$ , it is possible to drive the error rate  $\varepsilon \rightarrow 0$
- May require very long decoding delay / high complexity
- Practical capacity-approaching codes have been developed (eg Turbo codes, LDPC codes)

Cutoff Rate  $R_0$ :

- Bound for the attainable performance of convolutional codes
- Relates attainable code rate  $R$ , word error probability (reliability), and codeword length  $n$  as

$$\text{WER} < 2^{-n(R_0 - R)}$$

# Channel Capacity

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Instantaneous capacity in bits per complex dimension of an  $N$  tone BICM-OFDM system is given by

$$C(\mathbf{H}) = m - \frac{1}{N} \sum_{\ell=1}^m \sum_{i=1}^N \mathbb{E}_{b, Y_i} \left\{ \log_2 \left( \frac{\sum_{X_i \in \mathcal{X}} p(Y_i | \hat{H}_i, X_i)}{\sum_{X_i \in \mathcal{X}_b^\ell} p(Y_i | \hat{H}_i, X_i)} \right) \right\} \quad (1)$$

- $m$  is the number of bits per symbol (2 in our case)
- $\mathcal{X}$  is the signal constellation (QPSK in our case)
- $\mathcal{X}_b^\ell$  is the set of all constellation points  $X \in \mathcal{X}$  whose label has the value  $b \in \{0, 1\}$  in position  $\ell$
- $p(Y_i | \hat{H}_i, X_i)$  is the pdf of the channel output  $Y_i$  for given input  $X_i$  and channel estimate  $\hat{H}_i$ ,
- $\mathbb{E}_z\{\cdot\}$  denotes expectation with respect to  $z$ .

Instantaneous cutoff rate in bits per complex dimension

$$R_0(\mathbf{H}) = m(1 - \log_2(B(\mathbf{H}) + 1)) \quad (2)$$

with the instantaneous Bhattacharya parameter ( $\bar{b}$  denotes the complement of  $b$ )

$$B(\mathbf{H}) = \frac{1}{mN} \sum_{\ell=1}^m \sum_{i=1}^N \mathbb{E}_{b, Y_i} \left\{ \sqrt{\frac{\sum_{X_i \in \mathcal{X}_{\bar{b}}^\ell} p(Y_i | \hat{H}_i, X_i)}{\sum_{X_i \in \mathcal{X}_b^\ell} p(Y_i | \hat{H}_i, X_i)}} \right\}. \quad (3)$$

## A Note on Evaluating Capacity and Cutoff Rate

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- Analytical solution of the  $C(\mathbf{H})$ ,  $R_0(\mathbf{H})$  expectations not feasible
- We resort to Monte Carlo methods to evaluate (numerically) the capacity and cutoff rate as given above

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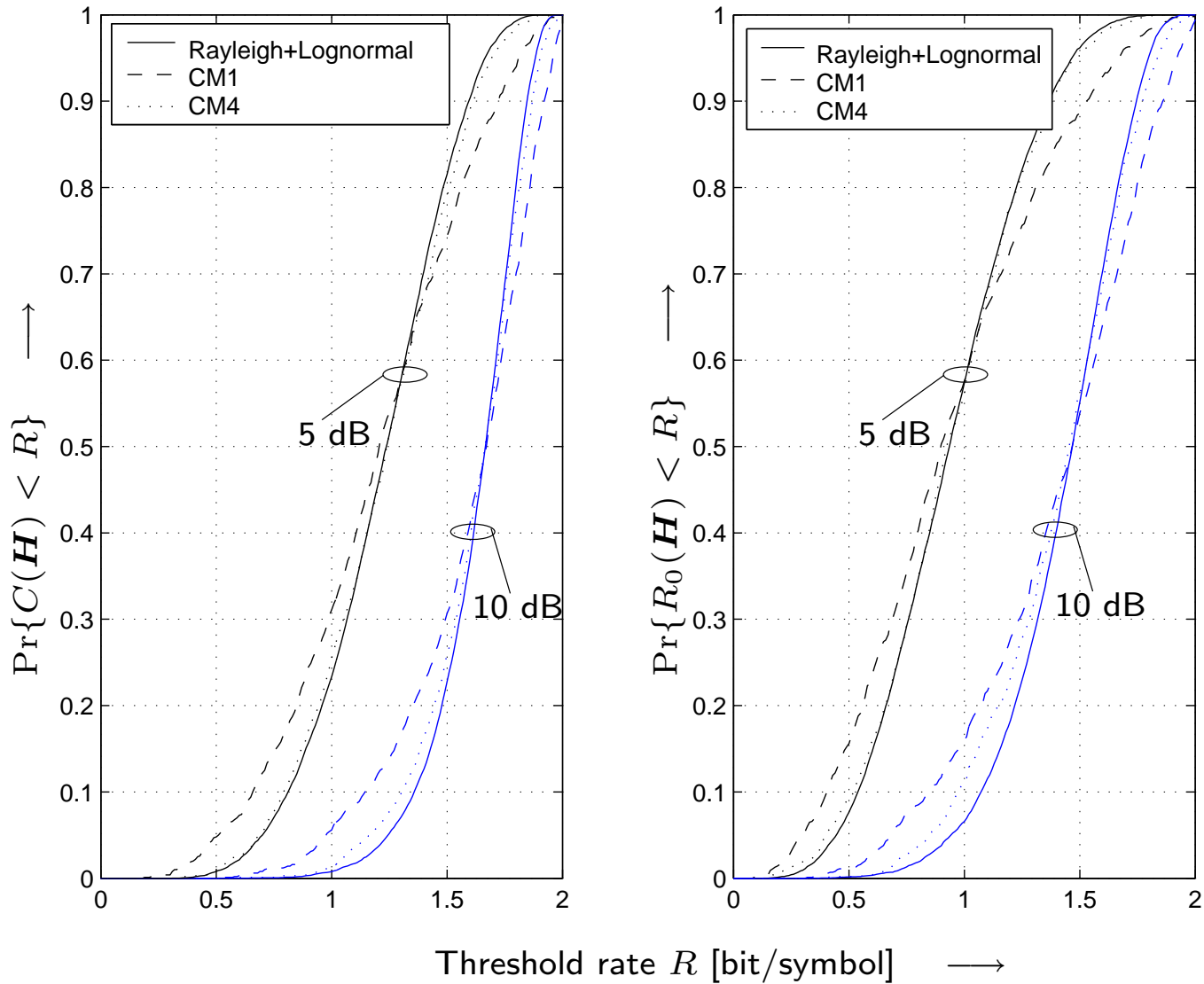
## Results

# Parameters

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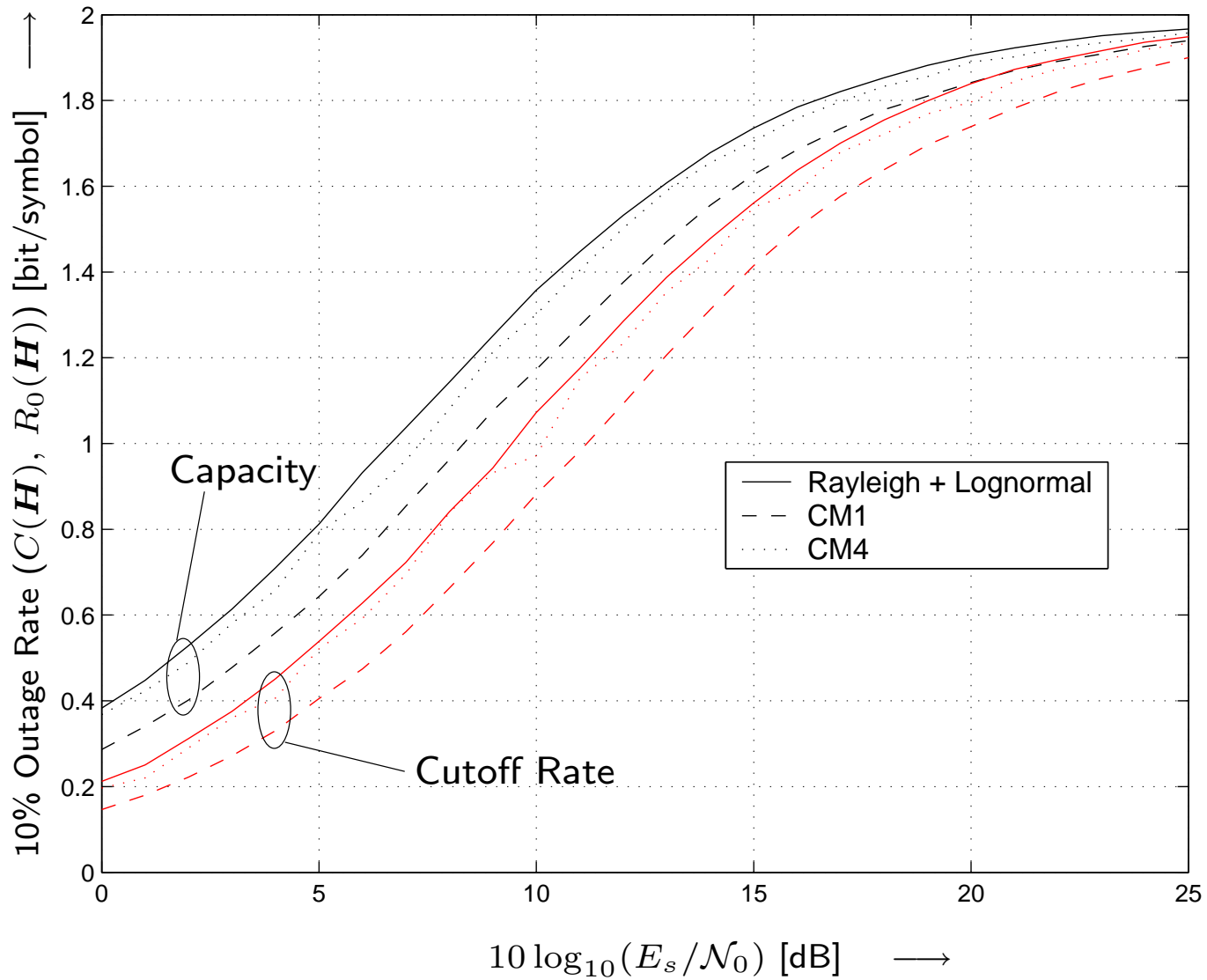
- Capacities and cutoff rates evaluated by Monte Carlo simulation using 1000 realizations of each channel model
- Simulation results using 100 realizations of each channel model
- Simulation results for 4 of the 10 modes of MB-OFDM proposal
- We only show CM1 and CM4 as they are the extreme cases
- Show as comparison “Rayleigh+Lognormal” (i.i.d Rayleigh fading on each carrier with an outer lognormal as with UWB)
- Channel estimation factor  $\eta = 0.25$  for imperfect CSI simulations

# Capacity and Cutoff CDFs — Perfect CSI



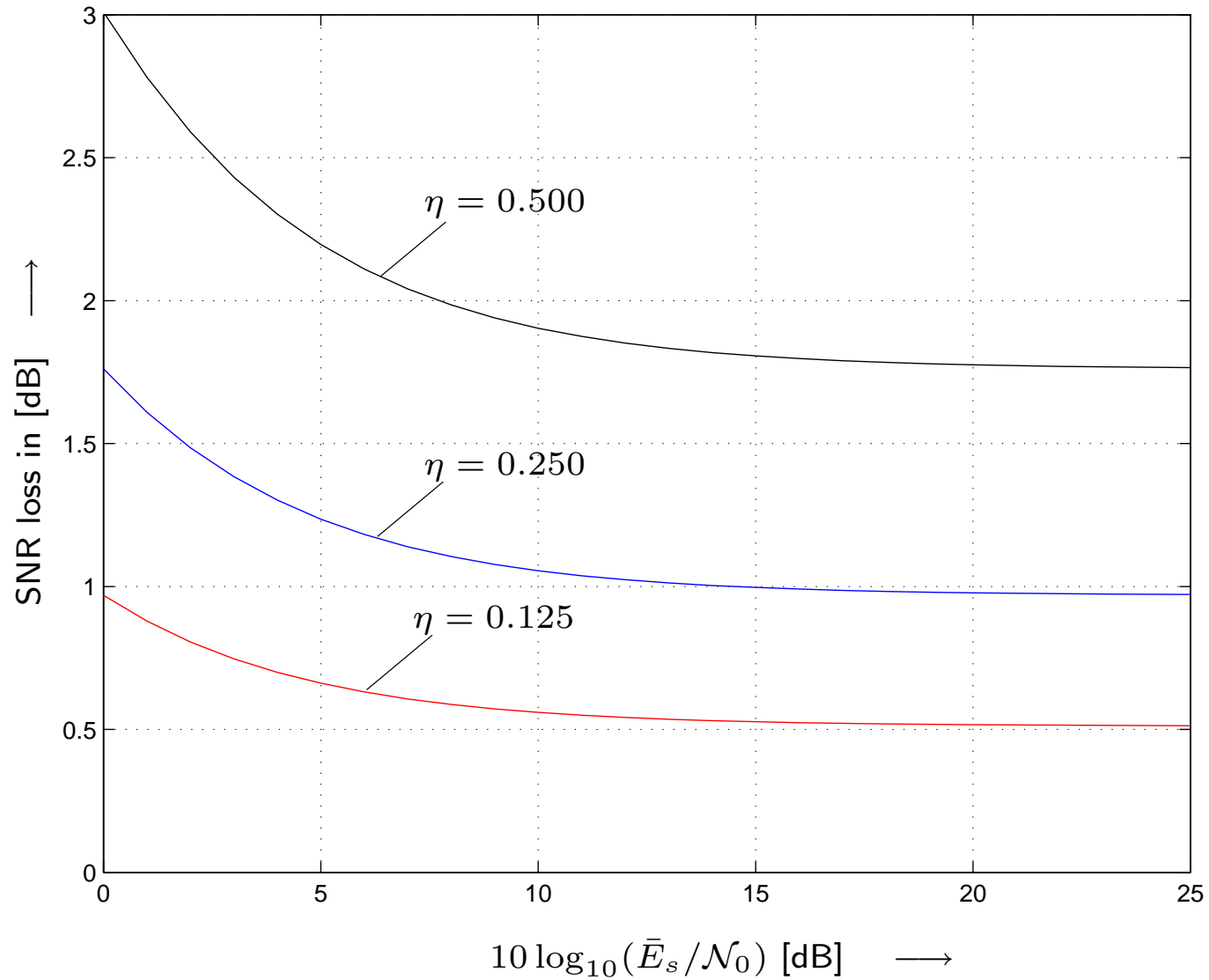
- Capacity and cutoff rate for UWB channel similar to i.i.d. Rayleigh fading channel with additional shadowing
- High diversity provided by UWB channel results in relatively steep outage curves  $\rightarrow$  reliability can be considerably improved by deliberately introducing coding redundancy (slightly more so for capacity i.e. powerful codes)
- Shadowing, which cannot be averaged out by coding, causes a flattening towards low outage probabilities  $\leq 0.1$
- In high outage probability range CM1 is superior to CM4, which corresponds to the large dominant eigenvalues

# 10% Outage Capacity and Cutoff Rate — Perfect CSI



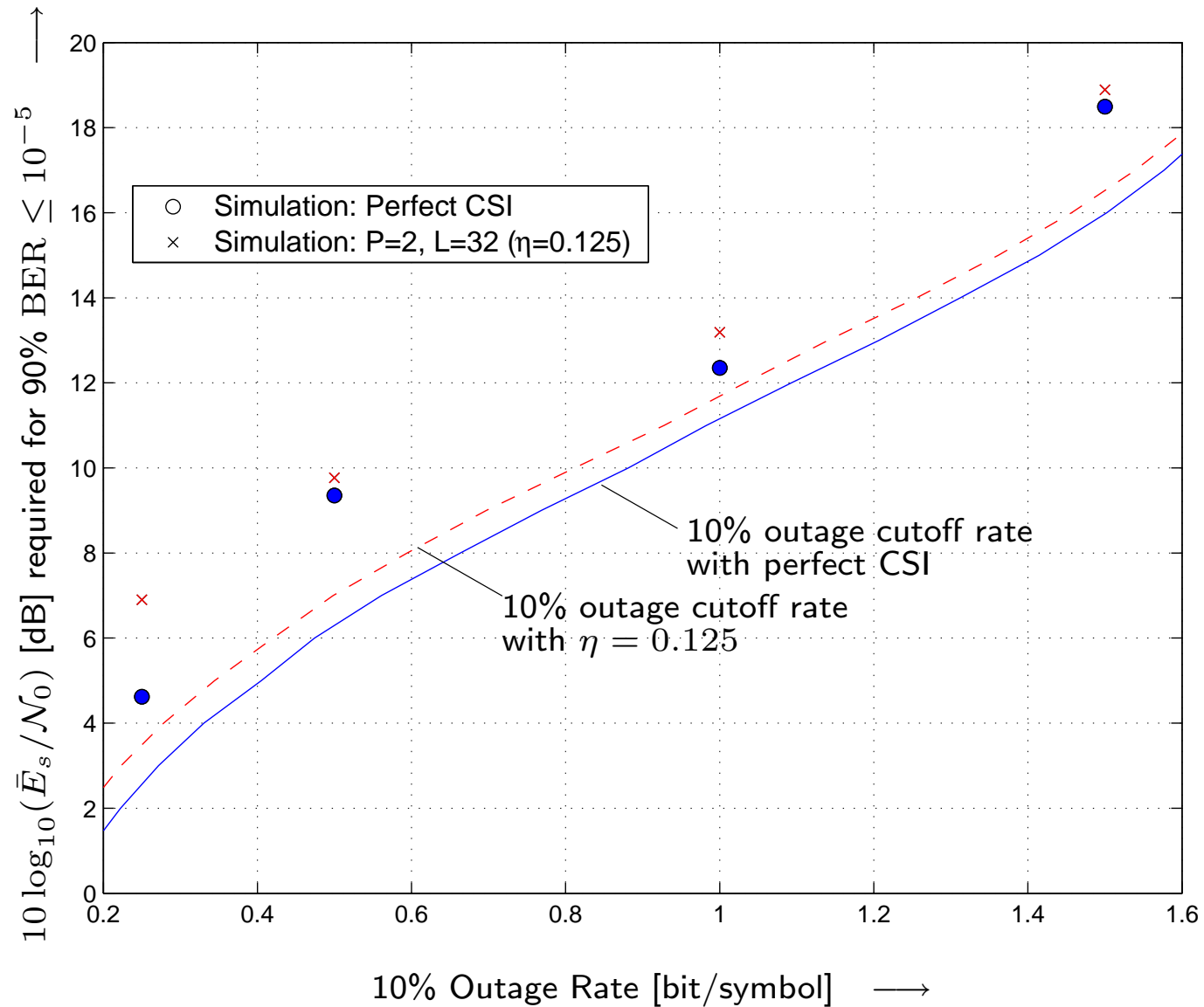
- Differences of 1-2 dB in power efficiency due to different UWB channel types, with CM4 providing basically the same performance as Rayleigh+Lognormal
- Comparison of capacity with cutoff rate curves → gains of 2.5 dB to 3 dB can be anticipated by the application of more powerful capacity approaching codes (eg: Turbo codes, LDPC codes)

# Loss in SNR due to Imperfect CSI



- Penalty due to imperfect CSI about 0.5 dB in the range of interest for the Multiband OFDM system.
- Reducing channel estimation overhead to  $P = 1$  ( $\eta = 0.25$ ) could be an interesting alternative for short packets, as the additional loss is only about 0.5 dB in  $\bar{E}_s/\mathcal{N}_0$
- Further reduction of pilot tones is not advisable as the gains in throughput are outweighed by the losses in power efficiency.

# 10% Outage Cutoff Rate vs Simulations



## 10% Outage Cutoff Rate vs Simulations — Analysis

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- Simulated SNR points well approach the cutoff-rate curves, which
  - justifies the relevance of the information-theoretic measure
  - confirms the coding approach used in Multiband OFDM
- Diversity provided by the UWB channel effectively exploited by the convolutional coding and interleaving scheme
- System with LSE channel estimation performs within 0.5 dB of the perfect CSI case (as expected from the cutoff-rate analysis)

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## Conclusions

## Conclusions (1)

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- UWB channel model is seen by OFDM systems as “fast” frequency-nonselctive Rayleigh fading, with additional shadowing
- The codes used in Multiband OFDM can capture all the diversity they can use with the 528 MHz bandwidth proposed
- Information-theoretic limits of UWB channel similar to those of a perfectly-interleaved Rayleigh channel plus lognormal shadowing

## Conclusions (2)

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- BICM-OFDM scheme proposed in Multiband OFDM performs close to the cutoff rate — well suited to exploit the available diversity
- Simple LSE channel estimation can perform within 0.5 dB of perfect CSI

Future work: we will consider the application of powerful codes (eg Turbo, LDPC) as well as modulation diversity, other modulation schemes, MIMO, comparisons with DS-UWB...

**Questions?**