Coded Multiband OFDM with Tone Interference

or, “How to Analyze Coded OFDM Error Rates in Quasi-Static Channels”

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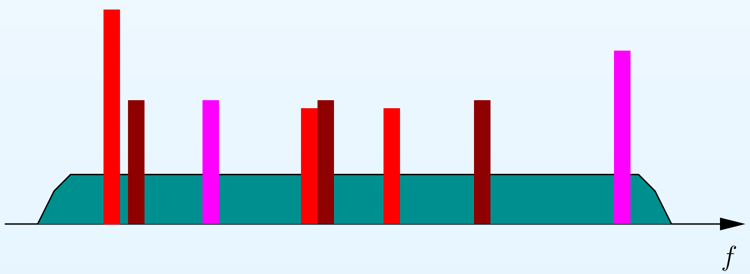
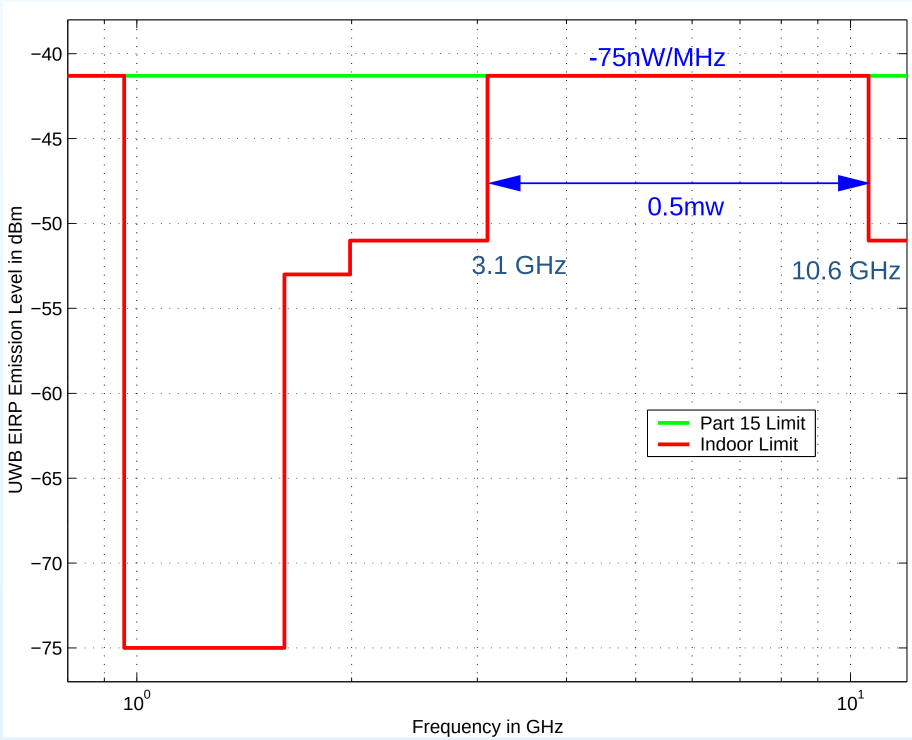
Agenda

- What is UWB?
- Multiband OFDM proposal for high data-rate UWB
- Performance Analysis of coded OFDM systems
  - Per-realization BER analysis
  - Direct average BER analysis
- Performance results for Multiband OFDM
  - without interference
  - with interference
- Conclusions

Ultra-Wideband (UWB) Wireless Transmission

- UWB is a wireless technology that uses an ultra-wide bandwidth ($>$ 500 MHz) and operates at very low power
- Underlay (reuse spectrum, potential for interference)
- FCC: license-exempt operation in 3.1–10.6 GHz band

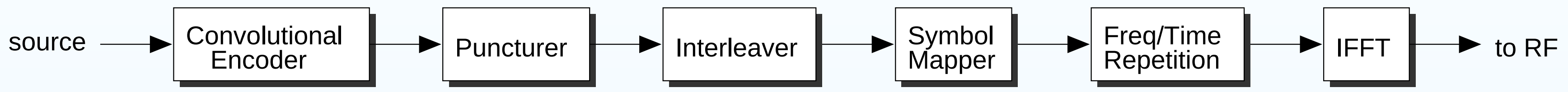




Multiband OFDM: Overview

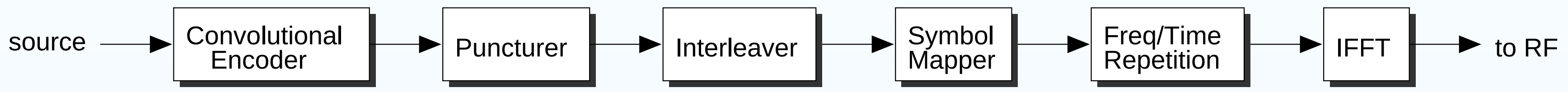
- Multiband OFDM is a leading proposal for high-rate UWB communication
- Multiband OFDM Alliance (MBOA) formed in 2003, now with over 170 member companies (Intel, Texas Instruments, . . .)
- Standardization of Multiband OFDM by
  - ~~IEEE TG 802.15.3a~~ (high-rate WPANs)
  - ECMA-368 High Rate Ultra Wideband PHY and MAC Standard (December 2005)
- Will be used in (for example)
  - Wireless USB
  - Wireless 1394 (Firewire)

Multiband OFDM: System Model



Transmitter

Multiband OFDM: System Model



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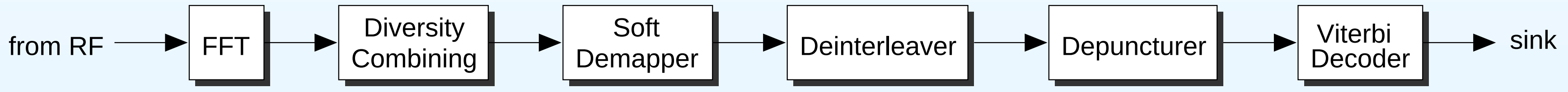

Receiver

Multiband OFDM: 802.15.3a Channel Model

- Impulse response consists of clusters of multipath components
- The channel is quasi-static
- The channel stays constant for (at least) one packet transmission
- We can only code over one realization of the channel
- (Also turns out the frequency-domain channel gains are Rayleigh — will be important later)

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Outage BER is a more relevant measure for quasi-static channels, since the channel may be constant for a long time...
BUT, it is hard to determine.

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- k/n convolutional code (rate R_c)
- d_{free} = free distance
- β_d = total number of bit errors associated with paths of Hamming weight d

Classical Average BER Analysis

What is the “standard” method for analyzing coded modulation?

- k/n convolutional code (rate R_c)
- d_{free} = free distance
- β_d = total number of bit errors associated with paths of Hamming weight d
- Average BER is given by the union bound

$$\text{BER} \leq \frac{1}{k} \sum_{d=d_{\text{free}}}^{\infty} \beta_d P_2(d)$$

  where $P_2(d)$ is the pairwise error probability (PEP) of two sequences differing in d positions
- Can get expressions for $P_2(d)$ in different types of channels (AWGN, fading, etc)

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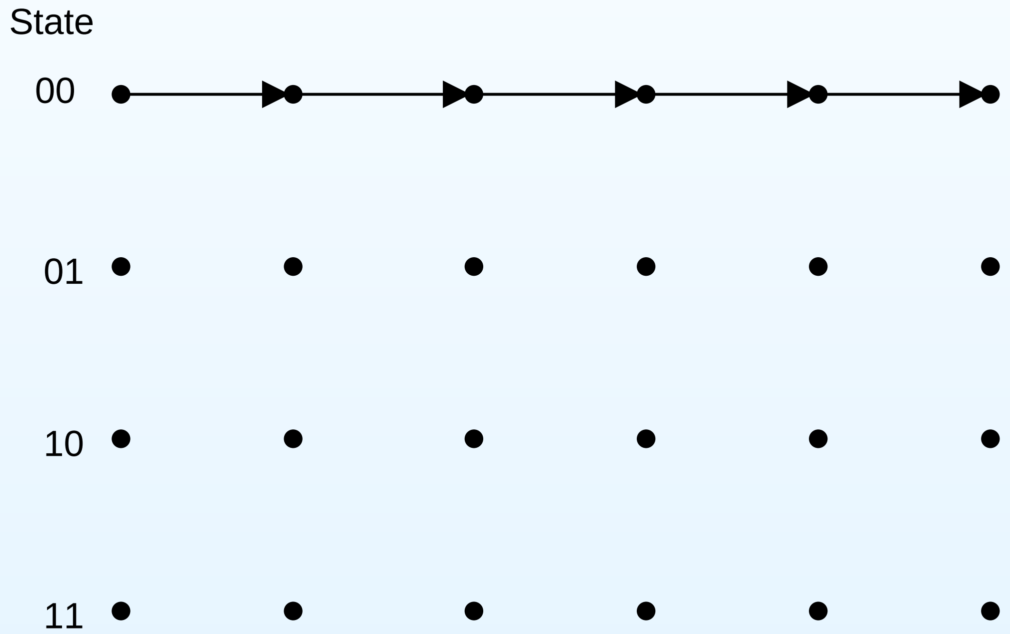
- Integrating over the pdf of channel distribution is NOT the thing to do here
- Question: if we know channel gains, can we predict the BER?
- Answer: Yes. Let’s examine the trellis of the code...

Error Vectors

Trellis representation of $R_c = 1/2$ (7,5) convolutional code

An example error path:



Input: $\{\;,\;\;,\;\;,\;\}$

Output e_1: $\{\;,\;\;,\;\;,\;\;,\;\;,\;\;,\;\;,\;\}$

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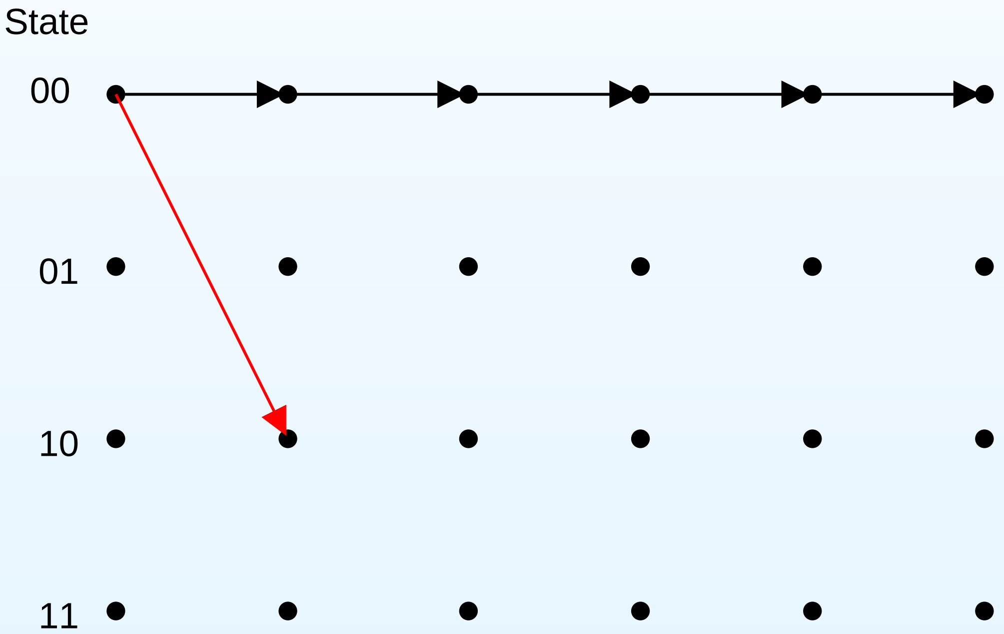

Input: $\{1, \;, \;, \;\}$

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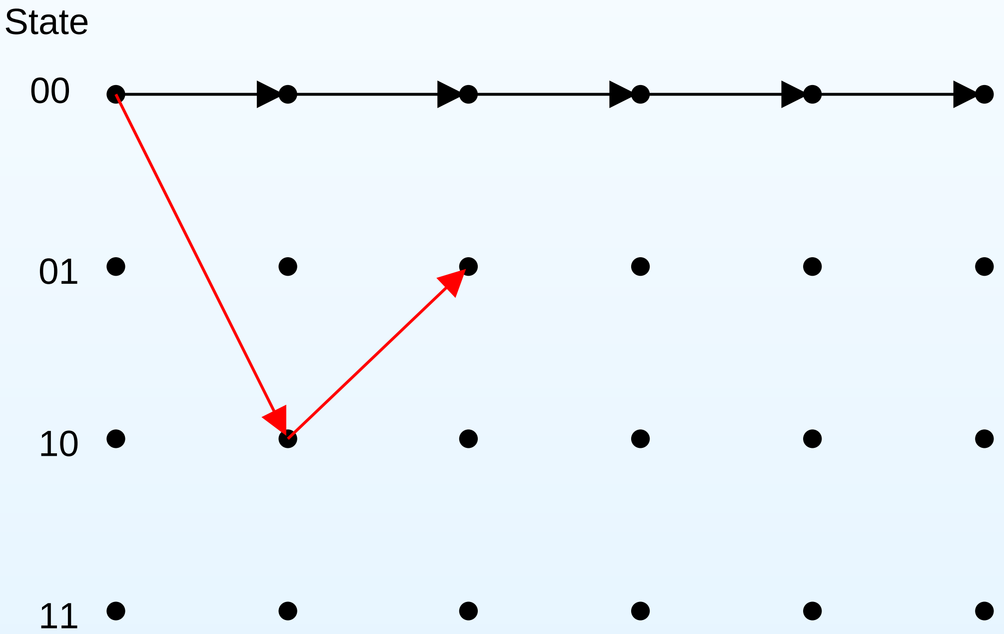

Input: $\{1, {\color{red}0},\ ,\ \}$

Output $\boldsymbol{e}_1$: $\{1, 1, {\color{red}1}, {\color{red}0},\ ,\ ,\ ,\ \}$

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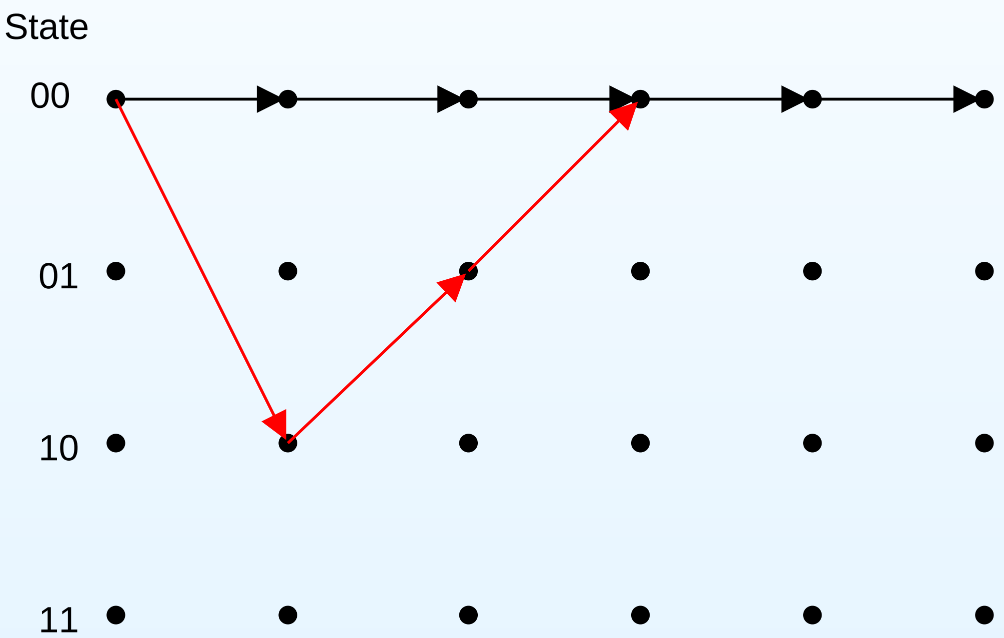

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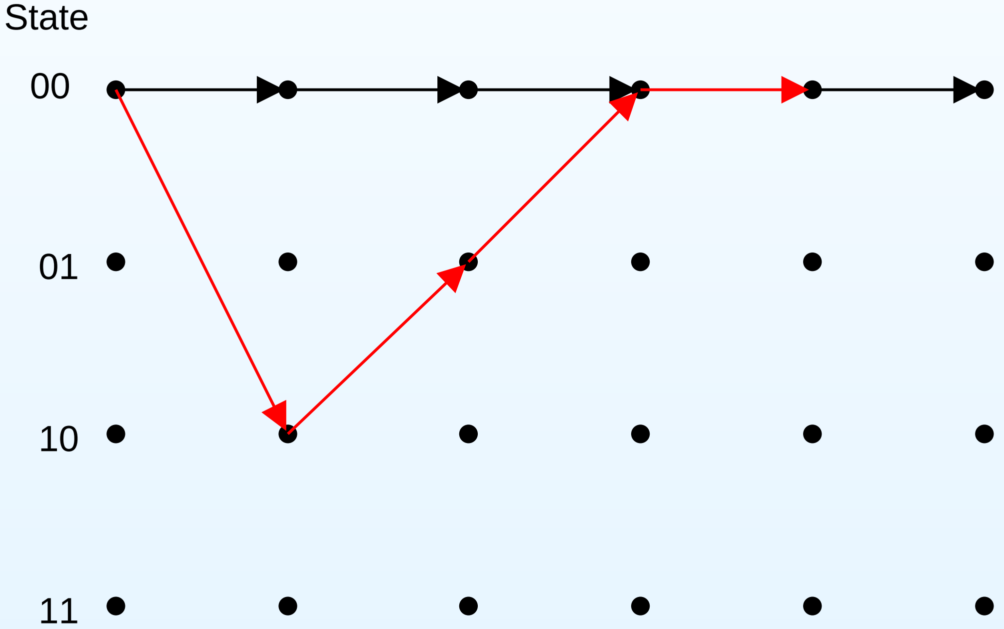

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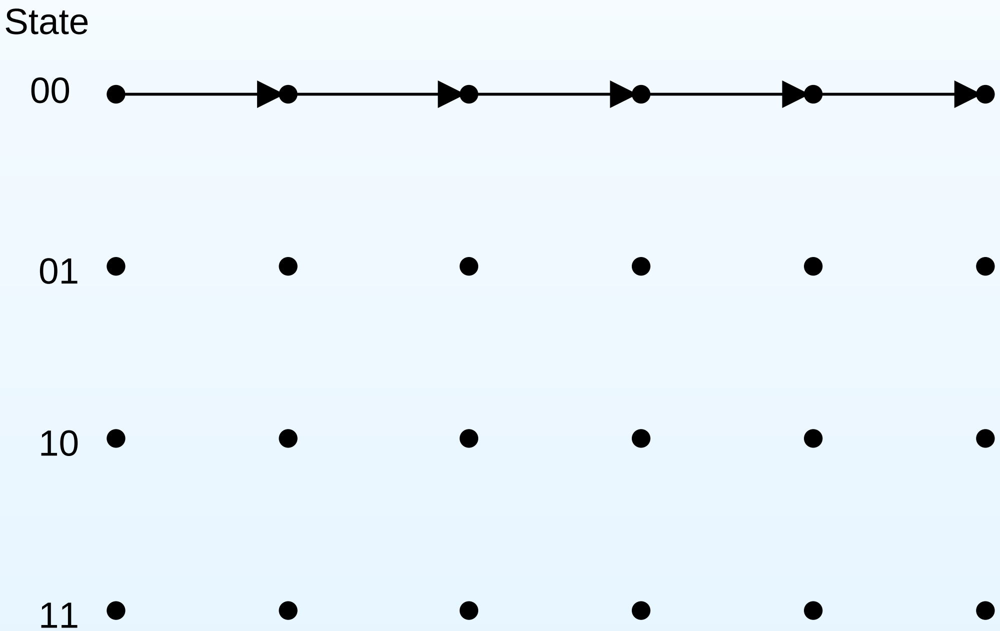

Input: $\{\ ,\ ,\ ,\ \}$

Output e_2: $\{\ ,\ ,\ ,\ ,\ ,\ ,\ ,\ \}$

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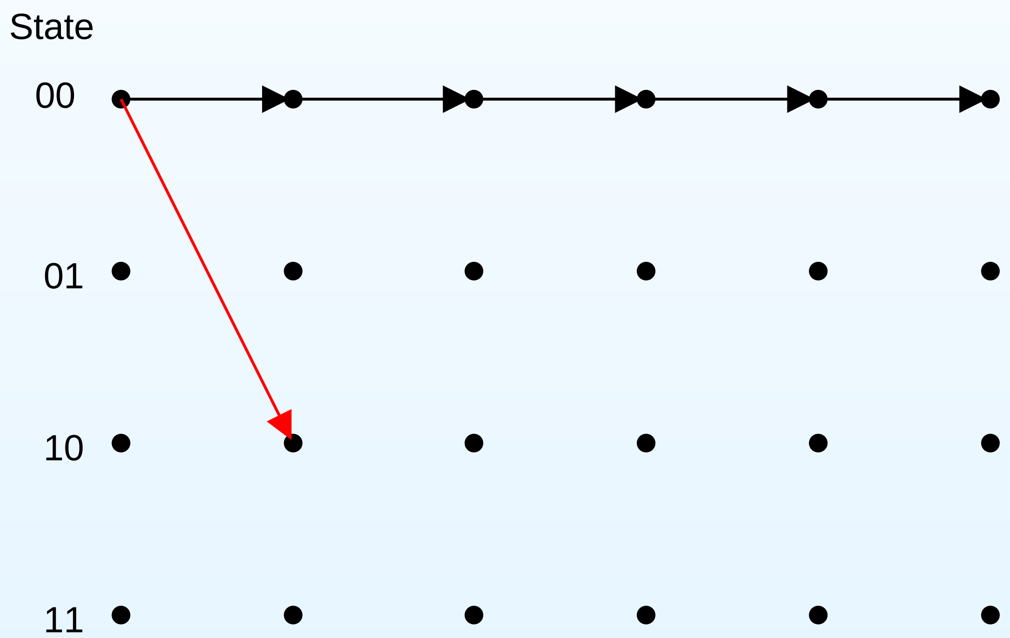

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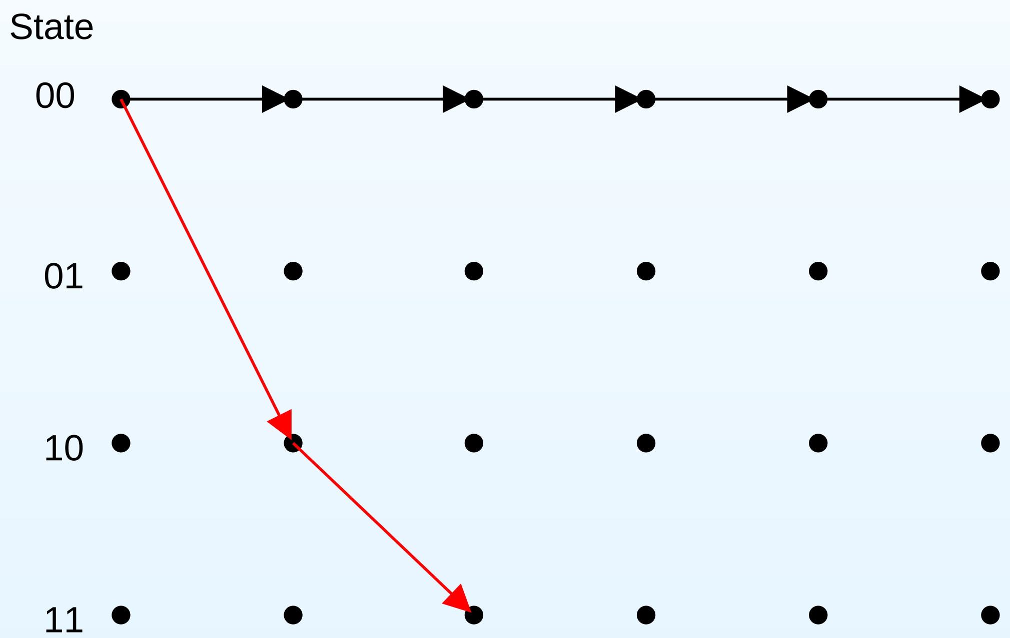

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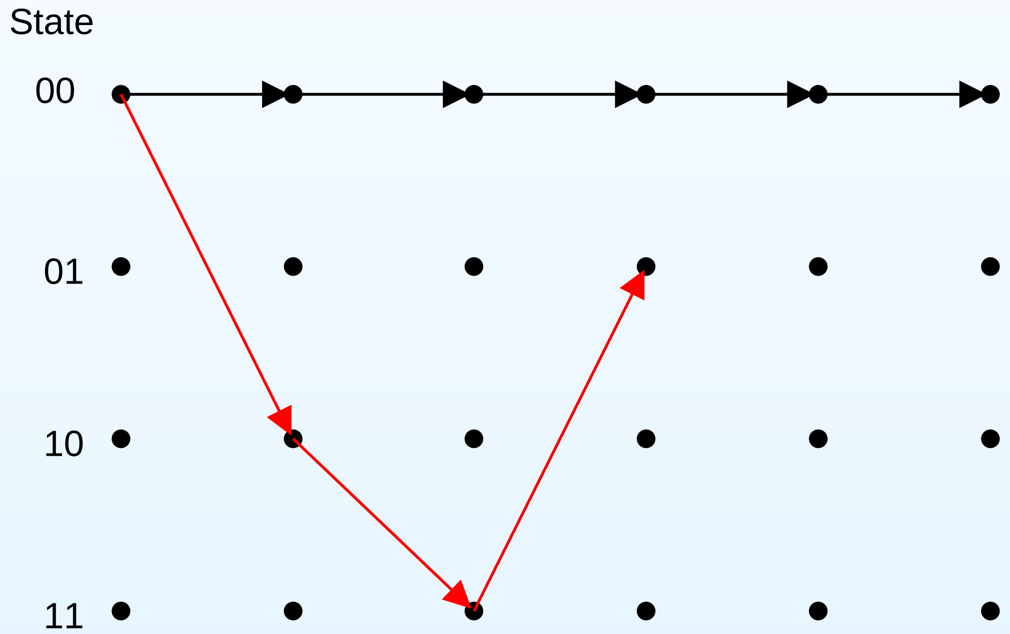

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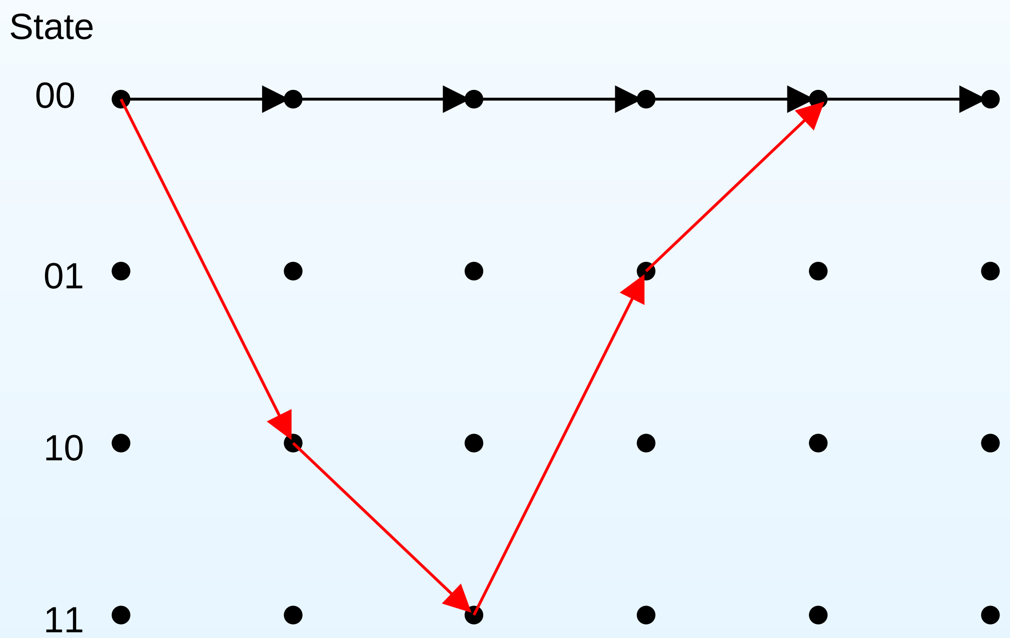

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$\mathcal{E}$ contains all the "most likely" error events (the ones with shortest length)

System Model

We consider the following frequency-domain system model

$$\boldsymbol{r} = \boldsymbol{H}\boldsymbol{x} + \boldsymbol{J} + \boldsymbol{n}$$

where

- $\boldsymbol{x}$ are the transmitted symbols
- $\boldsymbol{H} = \mathrm{diag}(\boldsymbol{h})$ is a (diagonal) matrix of channel gains
- $\boldsymbol{J}$ is the interference (frequency-domain)
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Interference is modelled as

$$i(t) = \sum_{k=1}^{N_i} \alpha_k e^{j(2\pi f_k t + \phi_k)}$$

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$$\boldsymbol{q}_{i,\ell} = [\underbrace{0 \;\; 0 \;\; \ldots \;\; 0}_{i-1} \;\; \underbrace{\boldsymbol{e}_\ell}_{l_\ell} \;\; \underbrace{0 \;\; 0 \;\; \ldots \;\; 0}_{L_c - l_\ell - i + 1}]^T$$

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- Now, given $\boldsymbol{q}_{i,\ell}$, the competing codeword is given by

$$\boldsymbol{v}_{i,\ell} = \boldsymbol{c} \oplus \boldsymbol{q}_{i,\ell}$$

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- true codeword $\boldsymbol{c} \longrightarrow \boldsymbol{x}$
- competing codeword $\boldsymbol{v}_{i,\ell} \longrightarrow \boldsymbol{z}_{i,\ell}$
- the PEP for the ℓth error vector starting in the ith position, i.e., the probability that $\boldsymbol{v}_{i,\ell}$ is detected given that $\boldsymbol{c}$ was transmitted, is given by

$$\mathrm{PEP}_{i,\ell}(\boldsymbol{H}, \boldsymbol{J}) = \Pr\left\{||\boldsymbol{r} - \boldsymbol{H}\boldsymbol{x}||^2 > ||\boldsymbol{r} - \boldsymbol{H}\boldsymbol{z}_{i,\ell}||^2 \;\middle|\; \boldsymbol{H}, \boldsymbol{J}\right\}$$

Method I: Per-realization BER

For given $\boldsymbol{H}, \boldsymbol{J}$ we can write

$$\mathrm{PEP}_{i,\ell}(\boldsymbol{H},\boldsymbol{J}) = Q\left(\frac{\frac{1}{2}||\boldsymbol{H}(\boldsymbol{x}-\boldsymbol{z}_{i,\ell})||^2 + \mathrm{Re}\left\{\boldsymbol{J}^H\boldsymbol{H}(\boldsymbol{x}-\boldsymbol{z}_{i,\ell})\right\}}{\sqrt{\frac{1}{2}\mathcal{N}_0||\boldsymbol{H}(\boldsymbol{x}-\boldsymbol{z}_{i,\ell})||^2}}\right)$$

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and if $\boldsymbol{J}=\boldsymbol{0}$, we can get

$$\mathrm{PEP}_{i,\ell}(\boldsymbol{H},\boldsymbol{J}) = Q\left(\sqrt{\frac{||\boldsymbol{H}(\boldsymbol{x}-\boldsymbol{z}_{i,\ell})||^2}{2\mathcal{N}_0}}\right)$$

Method I: Per-realization BER

BER for $\boldsymbol{e}_\ell$, starting in the ith position, is given by

$$P_{i,\ell}(\boldsymbol{H}, \boldsymbol{J}) = a_\ell \cdot \mathrm{PEP}_{i,\ell}(\boldsymbol{H}, \boldsymbol{J})$$

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Sum over all $\boldsymbol{e}_\ell \longrightarrow$ BER for the ith position

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Sum over all $\boldsymbol{e}_\ell \longrightarrow$ BER for the ith position

$$P_i(\boldsymbol{H},\boldsymbol{J}) = \sum_{\ell=1}^{L} P_{i,\ell}(\boldsymbol{H},\boldsymbol{J})$$

All positions equally likely $\longrightarrow$ BER $P(\boldsymbol{H},\boldsymbol{J})$ is

$$P(\boldsymbol{H},\boldsymbol{J}) = \frac{1}{L_c}\sum_{i=1}^{L_c} \min\left[\frac{1}{2}, \sum_{\ell=1}^{L} P_{i,\ell}(\boldsymbol{H},\boldsymbol{J})\right]$$

This is the (approximate) BER for a given $\boldsymbol{H}$

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Average BER:

$$P_{\text{avg}} = \frac{1}{N_c} \sum_{i=1}^{N_c} \mathbb{E}_{\boldsymbol{J}} \left\{ P(\boldsymbol{H}_i, \boldsymbol{J}) \right\}$$

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Average BER:

$$P_{\text{avg}} = \frac{1}{N_c} \sum_{i=1}^{N_c} \mathbb{E}_{\boldsymbol{J}} \{P(\boldsymbol{H}_i, \boldsymbol{J})\}$$

Outage BER:

$$P_{\text{out}} = \max_{\boldsymbol{H}_i \in \mathcal{H}_{\text{in}}} \mathbb{E}_{\boldsymbol{J}} \{P(\boldsymbol{H}_i, \boldsymbol{J})\}$$

(where $\mathcal{H}_{\text{in}}$ are the channels that are NOT in outage)

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- For average BER — slow!

To obtain average BER, we can do better for certain channel distributions

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If channel gains $\boldsymbol{h}$ are

- Rayleigh-distributed r.v.s, and
- Correlated with correlation matrix $\boldsymbol{\Sigma}_{\boldsymbol{hh}}$

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Start from PEP for position i, error vector ℓ

$$\mathrm{PEP}_{i,\ell} = \Pr\left\{ ||\boldsymbol{r} - \boldsymbol{H}\boldsymbol{x}||^2 > ||\boldsymbol{r} - \boldsymbol{H}\boldsymbol{z}_{i,\ell}||^2 \;\middle|\; \boldsymbol{H}, \boldsymbol{J} \right\}$$

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We only need to consider non-zero entries of $(\boldsymbol{x} - \boldsymbol{z}_{i,\ell}) \longrightarrow$ make

- $\boldsymbol{x}'$, $\boldsymbol{z}'_{i,\ell}$, $\boldsymbol{H}' = \mathrm{diag}(\boldsymbol{h}')$, $\boldsymbol{J}'$, and $\boldsymbol{n}'$
- $\boldsymbol{\Sigma_{h'h'}}$
- $\boldsymbol{D} = \mathrm{diag}(\boldsymbol{x}' - \boldsymbol{z}'_{i,\ell})$
- $\boldsymbol{g} = \boldsymbol{H}'(\boldsymbol{x}' - \boldsymbol{z}'_{i,\ell}) = \boldsymbol{D}\boldsymbol{h}'$

Method II: Direct Average BER

We have

$$\begin{aligned} \mathbb{E}(\boldsymbol{g}) &= \mathbf{0}_{\eta \times 1} \\ \mathbb{E}(\boldsymbol{g}\boldsymbol{g}^H) &= \boldsymbol{R}_{\boldsymbol{g}\boldsymbol{g}} = \boldsymbol{D}\boldsymbol{\Sigma}_{\boldsymbol{h}'\boldsymbol{h}'}\boldsymbol{D}^H \end{aligned}$$

i.e., the distribution of $\boldsymbol{g}$ is zero-mean complex Gaussian with covariance matrix $\boldsymbol{R}_{\boldsymbol{g}\boldsymbol{g}}$.

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Rewriting the PEP to include only contributing terms, we obtain

$$\begin{aligned}\overline{\text{PEP}}_{i,\ell} &= \Pr\left\{||\boldsymbol{r}' - \boldsymbol{H}'\boldsymbol{z}'_{i,\ell}||^2 - ||\boldsymbol{r}' - \boldsymbol{H}'\boldsymbol{x}'||^2 < 0\right\} \\ &= \Pr\left\{\boldsymbol{g}\boldsymbol{g}^H - \boldsymbol{g}(\boldsymbol{J}' + \boldsymbol{n}')^H - (\boldsymbol{J}' + \boldsymbol{n}')\boldsymbol{g}^H < 0\right\} \\ &= \Pr\left\{\Delta_{i,\ell}(\boldsymbol{D}) < 0\right\}\end{aligned}$$

Method II: Direct Average BER

$\Delta_{i,\ell}(\boldsymbol{D}) = \boldsymbol{y}^H \boldsymbol{A} \boldsymbol{y}$ and

$$\boldsymbol{y} = \begin{bmatrix} \boldsymbol{g} \\ \boldsymbol{J}' + \boldsymbol{n}' \end{bmatrix} \qquad \boldsymbol{A} = \begin{bmatrix} \boldsymbol{I}_\eta & -\boldsymbol{I}_\eta \\ -\boldsymbol{I}_\eta & \boldsymbol{0}_\eta \end{bmatrix}$$

and $\boldsymbol{y}$ has

- mean $\boldsymbol{\mu}_{\boldsymbol{y}\boldsymbol{y}}$
- covariance matrix $\boldsymbol{R}_{\boldsymbol{y}\boldsymbol{y}}$

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Laplace transform approach to get $\Pr\{\Delta_{i,\ell}(\boldsymbol{D}) < 0\}$

Method II: Laplace Transforms

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- non-faded interferers

$$\Phi_{i,\ell}(s) = \frac{\exp[-s\boldsymbol{\mu}_{\boldsymbol{yy}}^H(\boldsymbol{A}^{-1} + s\boldsymbol{R}_{\boldsymbol{yy}})^{-1}\boldsymbol{\mu}_{\boldsymbol{yy}}]}{\det(\boldsymbol{I}_{2\eta} + s\boldsymbol{R}_{\boldsymbol{yy}}\boldsymbol{A})}$$

Method II: Laplace Transforms

Laplace transforms of $\Delta_{i,\ell}(\boldsymbol{D})$ are

- non-faded interferers

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- iid Rayleigh interferers

$$\Phi_{i,\ell}(s) = \frac{1}{\det(\boldsymbol{I}_{2\eta} + s\boldsymbol{R}_{\boldsymbol{yy}}\boldsymbol{A})}$$

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- iid Rayleigh interferers

$$\Phi_{i,\ell}(s) = \frac{1}{\det(\boldsymbol{I}_{2\eta} + s\boldsymbol{R}_{\boldsymbol{yy}}\boldsymbol{A})}$$

and

$$\overline{\mathrm{PEP}}_{i,\ell} = \Pr\left\{\Delta_{i,\ell}(\boldsymbol{D}) < 0\right\} = \frac{1}{2\pi j}\int_{c-j\infty}^{c+j\infty} \Phi_{i,\ell}(s)\frac{\mathrm{d}s}{s}$$

Method II: Average BER

$\overline{\mathrm{BER}}$ for $\boldsymbol{e}_\ell$, starting in the ith position, is given by

$$\bar{P}_{i,\ell} = a_\ell \cdot \overline{\mathrm{PEP}}_{i,\ell}$$

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Sum over all $e_\ell \longrightarrow$ BER for the ith position

$$\bar{P}_i = \sum_{\ell=1}^{L} \bar{P}_{i,\ell}$$

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Sum over all $\boldsymbol{e}_\ell \longrightarrow$ BER for the ith position

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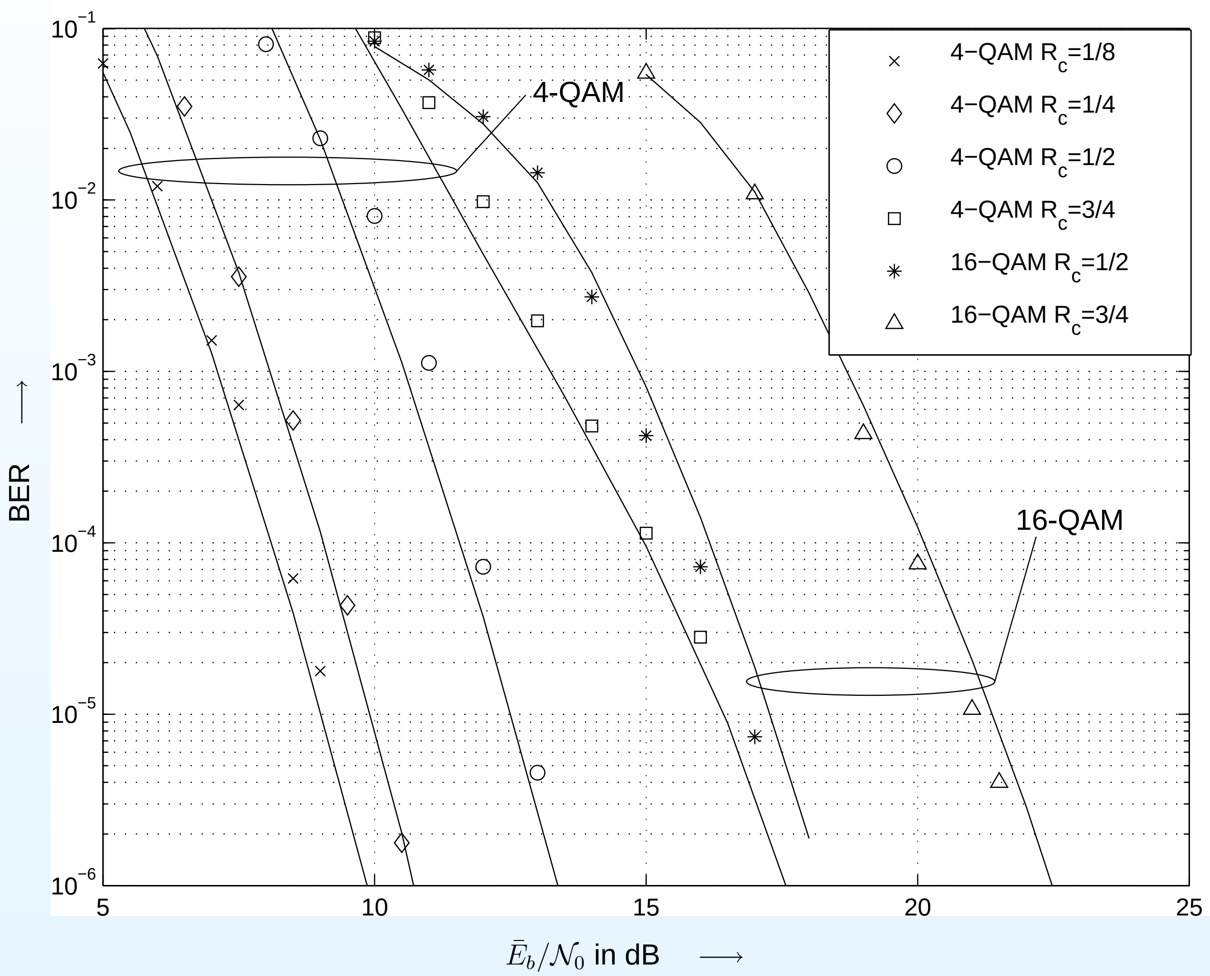
All positions equally likely $\longrightarrow \overline{\mathrm{BER}}$ is

$$\bar{P} = \frac{1}{L_c} \sum_{i=1}^{L_c} \bar{P}_i = \frac{1}{L_c} \sum_{i=1}^{L_c} \sum_{\ell=1}^{L} \bar{P}_{i,\ell}$$

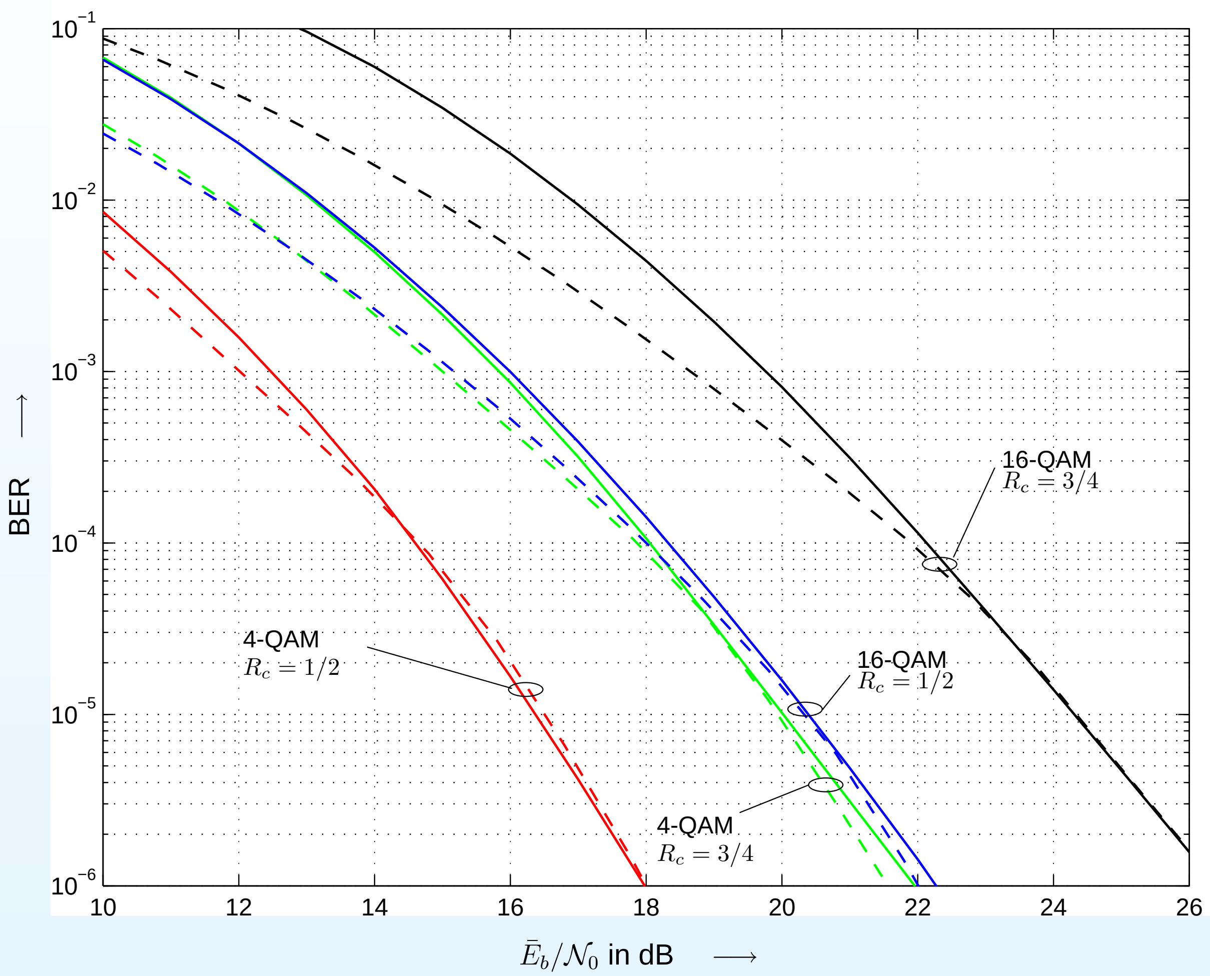
This is the (approximate) average BER

Results: Outage BER, no interference



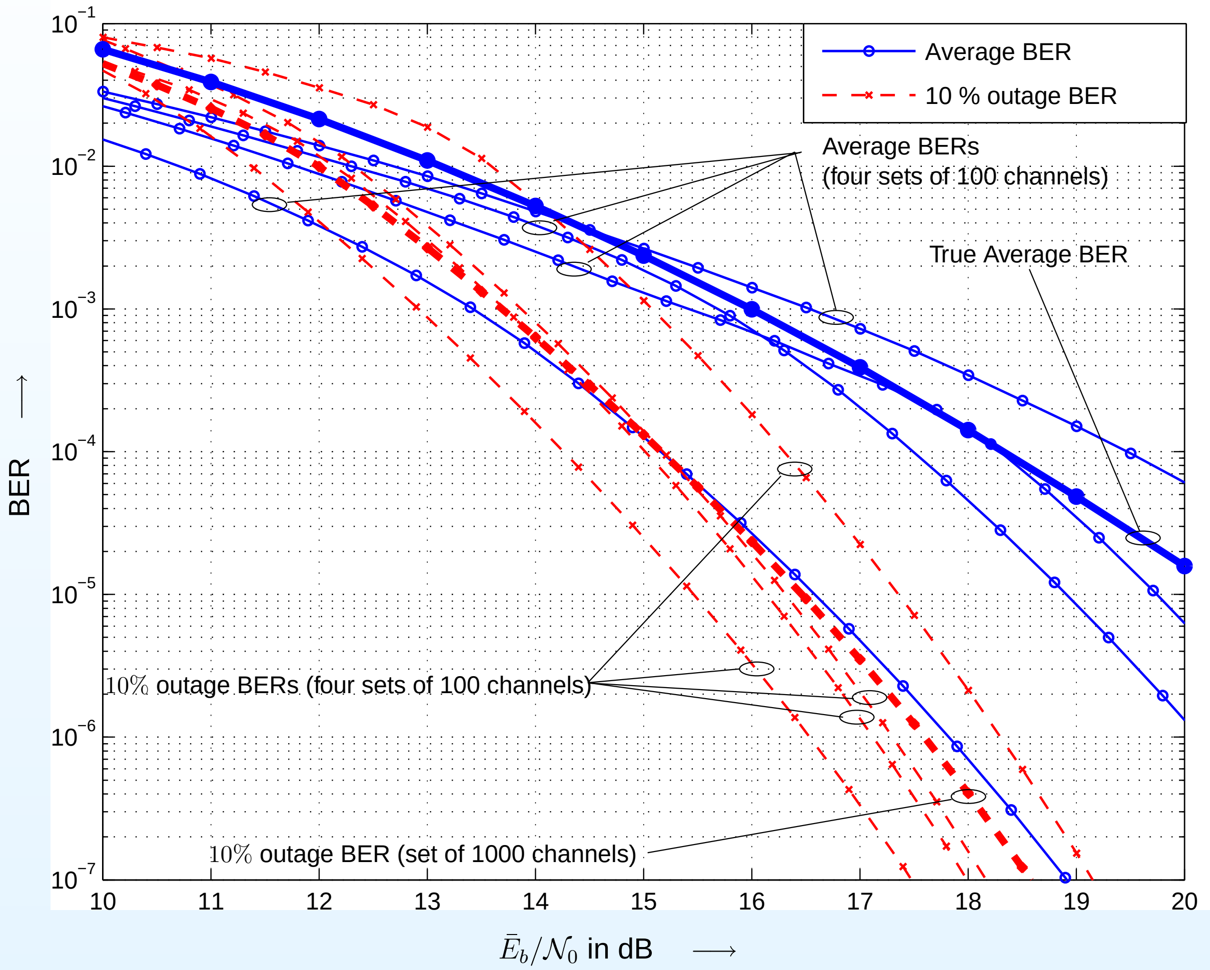
10% outage BER vs. $\bar{E}_b/\mathcal{N}_0$ from Method I (lines) and simulation results (markers) for different code rates and modulation schemes. UWB CM1 channel. Code rates $1/4$ and $1/8$ include repetition. No interference ($\boldsymbol{J} = \mathbf{0}_{N\times 1}$).

Results: Average BER, no interference



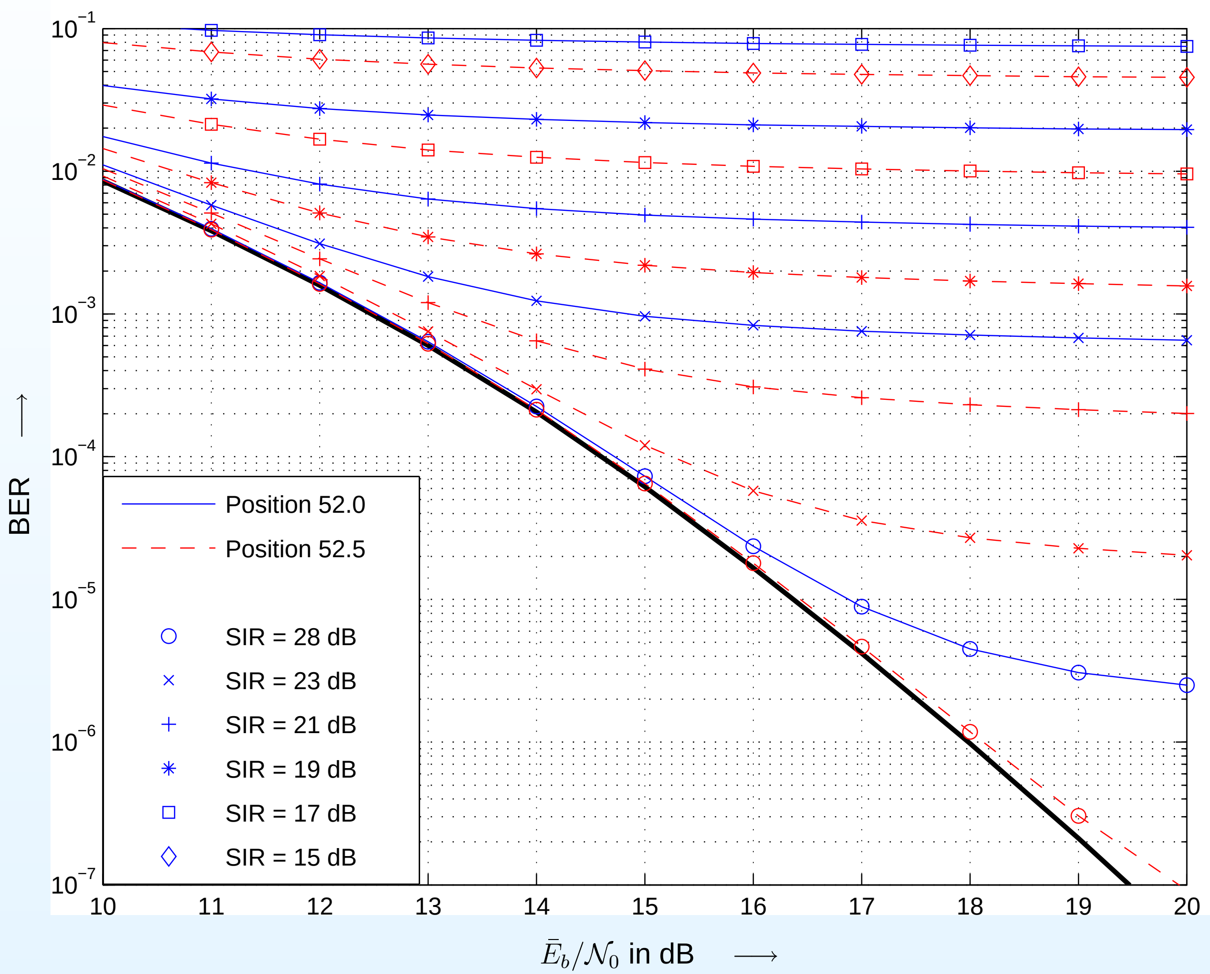
Average BER versus $\bar{E}_b/\mathcal{N}_0$ for 4-QAM and 16-QAM with code rates $R_c = 1/2$ and $3/4$. Solid lines: Direct average from Method II. Dashed lines: Method I with an average over 10,000 channel realizations. UWB CM1 channel. No interference ($\boldsymbol{J} = \mathbf{0}_{N\times 1}$).

Results: Variation in Realizations



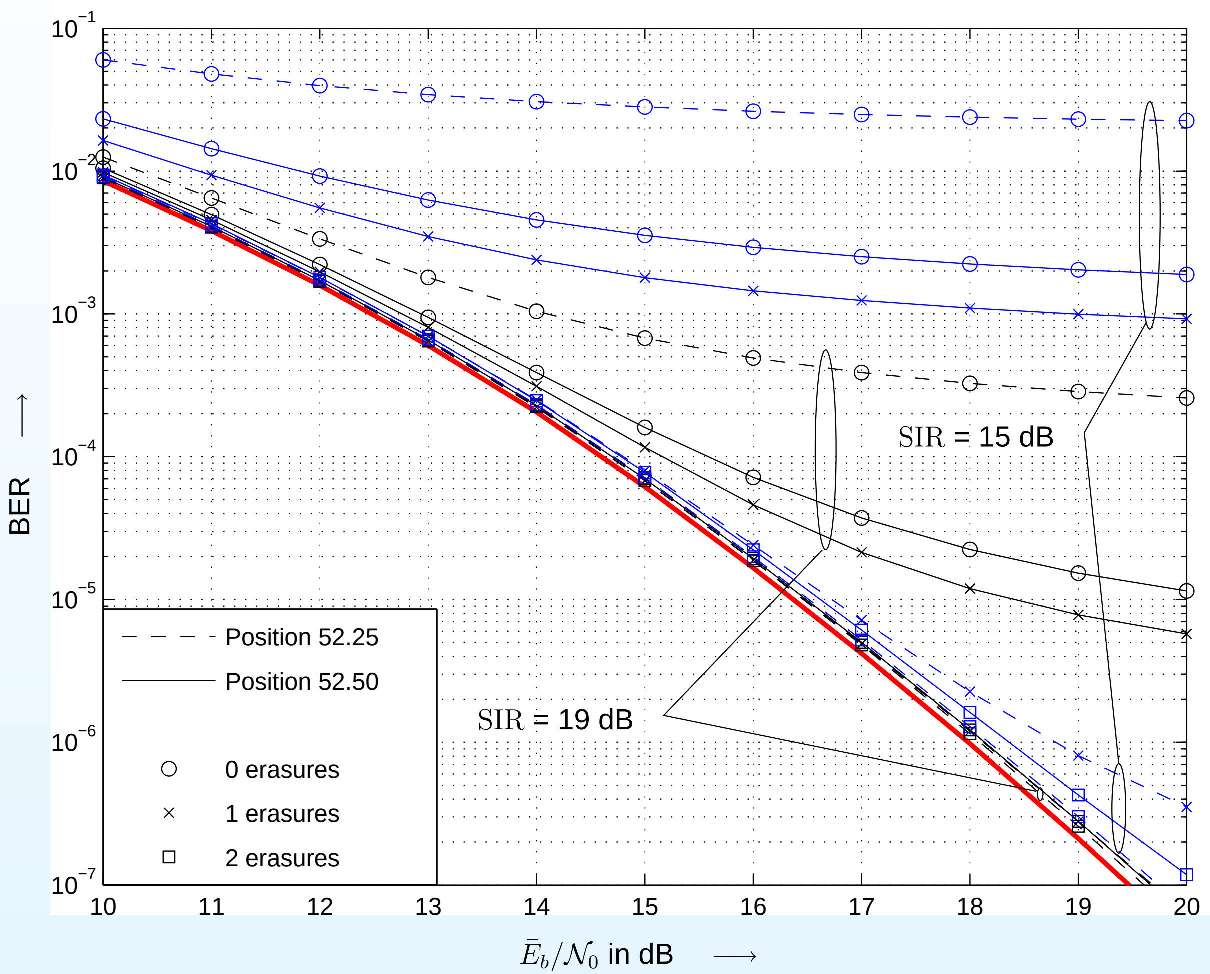
Average BER (solid lines) and 10% outage BER (dashed lines) versus $\bar{E}_b/\mathcal{N}_0$ for four different sets of 100 channels using Method I. UWB CM1 channel, $R_c = 1/2$, 16-QAM. No interference ($\boldsymbol{J} = \mathbf{0}_{N\times 1}$).

Results: Average BER, Rayleigh Interference



Average BER versus $\bar{E}_b/\mathcal{N}_0$ for various SIR with one Rayleigh-faded interferer, obtained using Method II. Interferer positions 52.0 (solid lines) and 52.5 (dashed lines). UWB CM1 channel, $R_c = 1/2$, 4-QAM.

Results: Erasures



Average BER versus $\bar{E}_b/\mathcal{N}_0$ for $\{0, 1, 2\}$ subcarrier erasures. One non-faded interferer, positions 52.25 (dashed lines) and 52.5 (dashed lines) and SIR = $\{15, 19\}$ dB, obtained using Method II. UWB CM1 channel, $R_c = 1/2$, 4-QAM.

Conclusions

- Multiband OFDM: an excellent technology for Wireless PANs
- We have analysis methods to estimate
  - Outage BER (per-realization)
  - Average BER (per-realization, and direct)
  - Effect of interference (both methods)
- These methods apply to any multi-carrier system in quasi-static channel
- Tone interference may have a strong impact on MB-OFDM
- We can use erasure decoding to improve performance

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