Impact of Tone Interference on Multiband OFDM

Chris Snow, Lutz Lampe and Robert Schober

Department of Electrical and Computer Engineering
University of British Columbia
Vancouver, Canada

ICUWB — 26 Sept 2006

Outline

An Alternate Title

"Approximate Error Rates of Coded OFDM in Quasi-Static Fading"

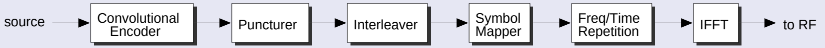
1. Multiband OFDM
   - System Model
   - Channel Model
   - Interference Model
2. Performance Analysis of Coded OFDM Systems
   - Error Vectors
   - Pairwise Error Probability
   - Per-realization BER Analysis
   - Average and Outage BER
3. Performance results for Multiband OFDM
   - Without Interference
   - With Interference

Multiband OFDM: Overview

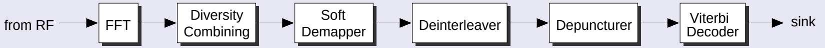
- Multiband OFDM is a leading high rate UWB system
- Standardization of Multiband OFDM by
  - ~~IEEE TG 802.15.3a for high-rate WPANs~~
  - ECMA-368 High Rate Ultra Wideband PHY and MAC Standard (December 2005)
- Example applications
  - Wireless USB
  - Wireless 1394 (Firewire)
- 3.1–10.6 GHz $\longrightarrow$ 14 subbands of 528 MHz
- First-gen: three subbands in 3.1–4.8 GHz

Multiband OFDM: System Model

Transmitter



Receiver



Multiband OFDM: Channel Model

Channel impulse response (802.15.3a model)

$$h(t) = X \sum_{l \geq 0} \sum_{k \geq 0} \alpha_{k,l} \delta(t - T_l - \tau_{k,l})$$

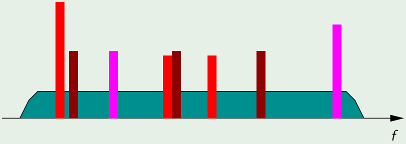
Impulse response consists of clusters of multipath components

- We will consider equivalent frequency-domain channel **H**
- Channel is quasi-static (constant for packet transmission)
- We can only code over one realization of the channel

Multiband OFDM: Interference Model

NBI in UWB

Spectral underlay $\longrightarrow$ many narrowband interferers



Interference Model

Model interference as a sum of N_i tone interferers

$$i(t) = \sum_{k=1}^{N_i} \alpha_k e^{j(2\pi f_k t + \phi_k)}$$

We consider frequency-domain sampled version

$$\mathbf{J} = \mathrm{DFT}\left([i(0) \quad i(T) \quad i(2T) \quad \ldots \quad i((N-1)T)]\right)$$

Analysis: Performance Measures

We will consider two different performance measures

Average Bit Error Rate (BER)

Average BER of a (large) number of different channel realizations

But, average BER is not too relevant for quasi-static channels...

Analysis: Performance Measures

We will consider two different performance measures

Average Bit Error Rate (BER)

Average BER of a (large) number of different channel realizations

But, average BER is not too relevant for quasi-static channels...

Outage BER

- if we assume $X\%$ of channels are bad ("in outage"), what is the worst-case performance of the remaining $(100 - X)\%$ of the channels?
- (we will consider $X = 10\%$ outage probability)

Outage BER is a more relevant measure for quasi-static channels...
BUT, it is hard to determine.

Analysis: BER for Quasi-static Channels

Average BER for Fast Fading Channel

- Get pairwise error probability $P_2(d|h)$ for given h
- Integrate over pdf of h to get $P_2(d)$, then apply union bound

$$\mathrm{BER} \leq \frac{1}{k} \sum_{d=d_{\mathrm{free}}}^{\infty} \beta_d P_2(d)$$

Analysis: BER for Quasi-static Channels

Average BER for Fast Fading Channel

- Get pairwise error probability $P_2(d|h)$ for given h
- Integrate over pdf of h to get $P_2(d)$, then apply union bound

$$\mathrm{BER} \leq \frac{1}{k} \sum_{d=d_{\mathrm{free}}}^{\infty} \beta_d P_2(d)$$

Quasi-static Channel

Each coded block transmitted over one channel realization

- Only "sees" a limited number of channel gains
- (and, correlation between bits depends on starting position)

CANNOT simply integrate over pdf of fading distribution!

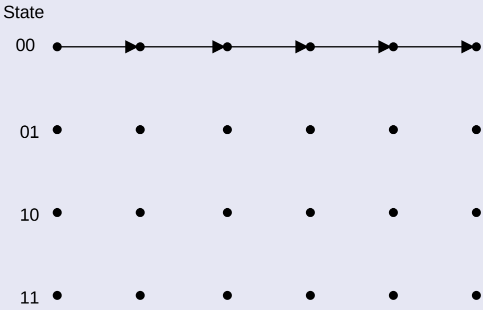
Question

Can we predict the BER of a given realization?

Analysis: Error Vectors

Trellis representation of $R_c = 1/2$ (7,5) convolutional code

An example error path

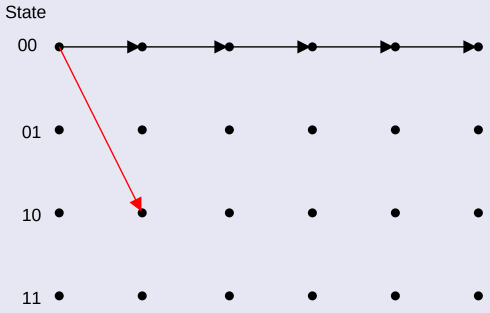


Input: $\{ \ , \ , \ , \ \}$
Output $\mathbf{e}_1$: $\{ \ , \ , \ , \ , \ , \ , \ , \ \}$

Analysis: Error Vectors

Trellis representation of $R_c = 1/2$ (7,5) convolutional code

An example error path



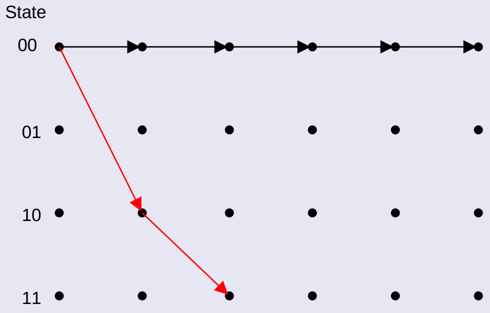
Input: $\{1, \ , \ , \ \}$

Output $\mathbf{e}_1$: $\{1, 1, \ , \ , \ , \ , \ , \ \}$

Analysis: Error Vectors

Trellis representation of $R_c = 1/2$ (7,5) convolutional code

An example error path



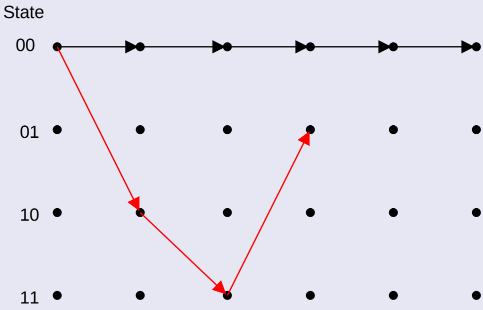
Input: $\{1, 1, \ , \ \}$

Output $\mathbf{e}_1$: $\{1, 1, 0, 1, \ , \ , \ , \ \}$

Analysis: Error Vectors

Trellis representation of $R_c = 1/2$ (7,5) convolutional code

An example error path



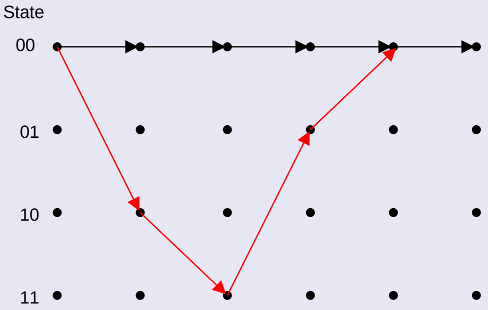
Input: $\{1, 1, 0, \ \}$

Output $\mathbf{e}_1$: $\{1, 1, 0, 1, 0, 1, \ , \ \}$

Analysis: Error Vectors

Trellis representation of $R_c = 1/2$ (7,5) convolutional code

An example error path



Input: $\{1, 1, 0, 0\}$
Output $\mathbf{e}_1$: $\{1, 1, 0, 1, 0, 1, 1, 1\}$

Analysis: Error Vectors

- Let's call $\mathbf{e}_\ell$ an "error vector" — it represents the output of a certain deviation from the all-zero path in the trellis

Analysis: Error Vectors

- Let's call $\mathbf{e}_\ell$ an "error vector" — it represents the output of a certain deviation from the all-zero path in the trellis
- Make the set of all error vectors $\mathbf{e}_\ell$ of less than a certain weight (L of them in total)
- Call this set $\mathcal{E} = \{\mathbf{e}_1, \ldots, \mathbf{e}_L\}$

Analysis: Error Vectors

- Let's call $\mathbf{e}_\ell$ an "error vector" — it represents the output of a certain deviation from the all-zero path in the trellis
- Make the set of all error vectors $\mathbf{e}_\ell$ of less than a certain weight (L of them in total)
- Call this set $\mathcal{E} = \{\mathbf{e}_1, \ldots, \mathbf{e}_L\}$
- l_ℓ is the length of $\mathbf{e}_\ell$
- a_ℓ is the number of information bit errors associated with $\mathbf{e}_\ell$

$\mathcal{E}$ contains all the "most likely" error events (the ones with shortest length)

Analysis: System Model

Frequency-domain System Model

$$\mathbf{r} = \mathbf{H}\mathbf{x} + \mathbf{J} + \mathbf{n}$$

where

- $\mathbf{x}$ are the transmitted symbols
- $\mathbf{H} = \mathrm{diag}(\mathbf{h})$ is a (diagonal) matrix of channel gains
- $\mathbf{J}$ is the interference (frequency-domain)
- $\mathbf{n}$ are AWGN noise variables

Analysis: PEP

- We transmit the true codeword $\mathbf{c}$ of length L_c (all zeros)

Analysis: PEP

- We transmit the true codeword **c** of length L_c (all zeros)
- We consider an error starting in position i of the codeword

Analysis: PEP

- We transmit the true codeword $\mathbf{c}$ of length L_c (all zeros)
- We consider an error starting in position i of the codeword
- For each error vector $\mathbf{e}_\ell$, we form the "full" error codeword (length L_c)

$$\mathbf{q}_{i,\ell} = [\underbrace{0 \;\; 0 \; \ldots \; 0}_{i-1} \;\; \underbrace{\mathbf{e}_\ell}_{l_\ell} \;\; \underbrace{0 \;\; 0 \; \ldots \; 0}_{L_c - l_\ell - i + 1}]^T$$

Analysis: PEP

- We transmit the true codeword $\mathbf{c}$ of length L_c (all zeros)
- We consider an error starting in position i of the codeword
- For each error vector $\mathbf{e}_\ell$, we form the "full" error codeword (length L_c)

$$\mathbf{q}_{i,\ell} = [\underbrace{0 \;\; 0 \;\; \ldots \;\; 0}_{i-1} \;\; \underbrace{\mathbf{e}_\ell}_{l_\ell} \;\; \underbrace{0 \;\; 0 \;\; \ldots \;\; 0}_{L_c - l_\ell - i + 1}]^T$$

- Now, given $\mathbf{q}_{i,\ell}$, the competing codeword is given by

$$\mathbf{v}_{i,\ell} = \mathbf{c} \oplus \mathbf{q}_{i,\ell}$$

Analysis: PEP

- We can map the two codewords into corresponding modulated symbols:

Analysis: PEP

- We can map the two codewords into corresponding modulated symbols:
- true codeword $\mathbf{c} \longrightarrow \mathbf{x}$

Analysis: PEP

- We can map the two codewords into corresponding modulated symbols:
- true codeword $\mathbf{c} \longrightarrow \mathbf{x}$
- competing codeword $\mathbf{v}_{i,\ell} \longrightarrow \mathbf{z}_{i,\ell}$

Analysis: PEP

- We can map the two codewords into corresponding modulated symbols:
- true codeword $\mathbf{c} \longrightarrow \mathbf{x}$
- competing codeword $\mathbf{v}_{i,\ell} \longrightarrow \mathbf{z}_{i,\ell}$
- the PEP for the ℓth error vector starting in the ith position, i.e., the probability that $\mathbf{v}_{i,\ell}$ is detected given that $\mathbf{c}$ was transmitted, is given by

$$\mathrm{PEP}_{i,\ell}(\mathbf{H}, \mathbf{J}) = \Pr\left\{ ||\mathbf{r} - \mathbf{H}\mathbf{x}||^2 > ||\mathbf{r} - \mathbf{H}\mathbf{z}_{i,\ell}||^2 \;\Big|\; \mathbf{H}, \mathbf{J} \right\}$$

Analysis: Per-realization BER

For given $\mathbf{H}, \mathbf{J}, i, \ell$

PEP

$$\mathrm{PEP}_{i,\ell}(\mathbf{H},\mathbf{J}) = Q\left(\frac{\frac{1}{2}||\mathbf{H}(\mathbf{x}-\mathbf{z}_{i,\ell})||^2 + \mathrm{Re}\left\{\mathbf{J}^H\mathbf{H}(\mathbf{x}-\mathbf{z}_{i,\ell})\right\}}{\sqrt{\frac{1}{2}\mathcal{N}_0||\mathbf{H}(\mathbf{x}-\mathbf{z}_{i,\ell})||^2}}\right)$$

PEP if $\mathbf{J} = \mathbf{0}$

$$\mathrm{PEP}_{i,\ell}(\mathbf{H},\mathbf{J}) = Q\left(\sqrt{\frac{||\mathbf{H}(\mathbf{x}-\mathbf{z}_{i,\ell})||^2}{2\mathcal{N}_0}}\right)$$

Analysis: Per-realization BER

BER for $\mathbf{e}_\ell$, starting in the ith position, is given by

$$P_{i,\ell}(\mathbf{H},\mathbf{J}) = a_\ell \cdot \mathrm{PEP}_{i,\ell}(\mathbf{H},\mathbf{J})$$

Sum over all $\mathbf{e}_\ell \longrightarrow$ BER for the ith position

$$P_i(\mathbf{H},\mathbf{J}) = \sum_{\ell=1}^{L} P_{i,\ell}(\mathbf{H},\mathbf{J})$$

All positions equally likely $\longrightarrow$ BER $P(\mathbf{H},\mathbf{J})$ is

$$P(\mathbf{H},\mathbf{J}) = \frac{1}{L_c}\sum_{i=1}^{L_c} \min\left[\frac{1}{2}, \sum_{\ell=1}^{L} P_{i,\ell}(\mathbf{H},\mathbf{J})\right]$$

This is the (approximate) BER for a given **H** and **J**

Analysis: Average and Outage BER

Once we know $P(\mathbf{H}, \mathbf{J})$, we can easily obtain both average and outage BER

Analysis: Average and Outage BER

Once we know $P(\mathbf{H}, \mathbf{J})$, we can easily obtain both average and outage BER

Average BER

$$P_{\text{avg}} = \frac{1}{N_c} \sum_{i=1}^{N_c} \mathbb{E}_{\mathbf{J}} \left\{ P(\mathbf{H}_i, \mathbf{J}) \right\}$$

Analysis: Average and Outage BER

Once we know $P(\mathbf{H}, \mathbf{J})$, we can easily obtain both average and outage BER

Average BER

$$P_{\text{avg}} = \frac{1}{N_c} \sum_{i=1}^{N_c} \mathbb{E}_{\mathbf{J}} \left\{ P(\mathbf{H}_i, \mathbf{J}) \right\}$$

Outage BER

$$P_{\text{out}} = \max_{\mathbf{H}_i \in \mathcal{H}_{\text{in}}} \mathbb{E}_{\mathbf{J}} \left\{ P(\mathbf{H}_i, \mathbf{J}) \right\}$$

(where $\mathcal{H}_{\text{in}}$ are the channels that are NOT in outage)

Results: Parameters

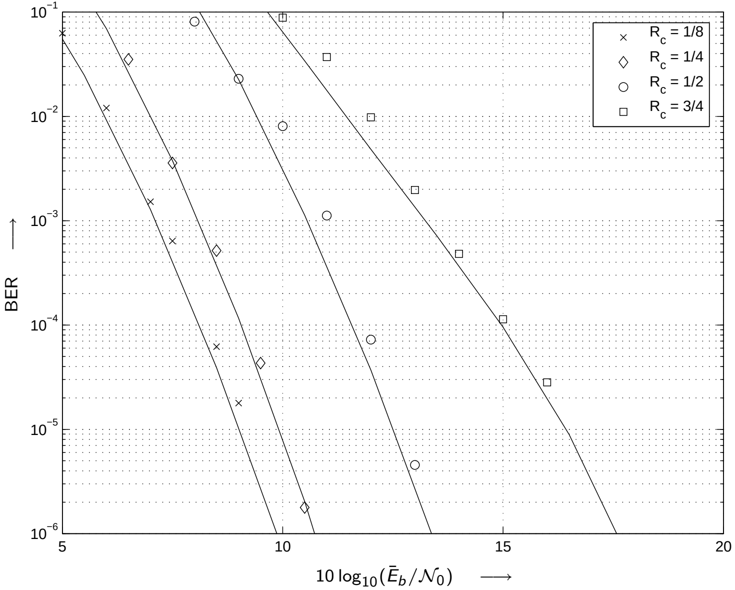
MB-OFDM System

- QPSK
- convolutional code
- AWGN and UWB CM1 channels

Narrowband Interferer

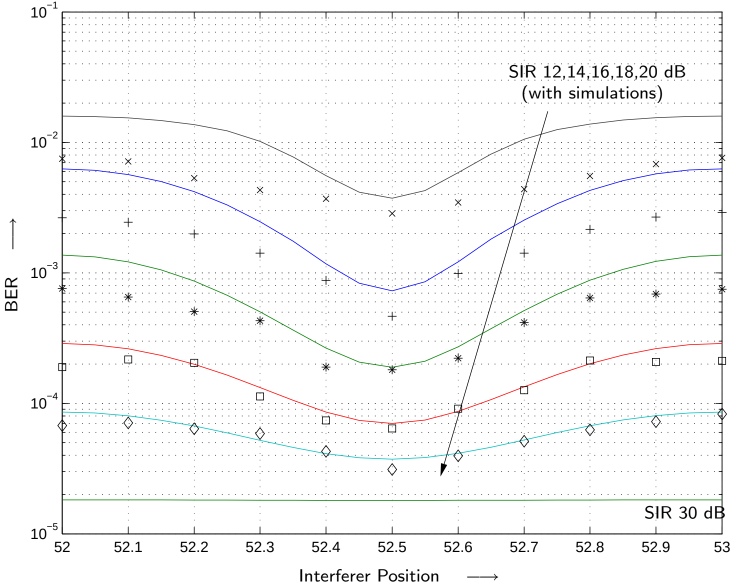
- One tone interferer
- f_1 varies position between two MB-OFDM subcarriers
- AWGN channel

Results: Outage BER, No Interference



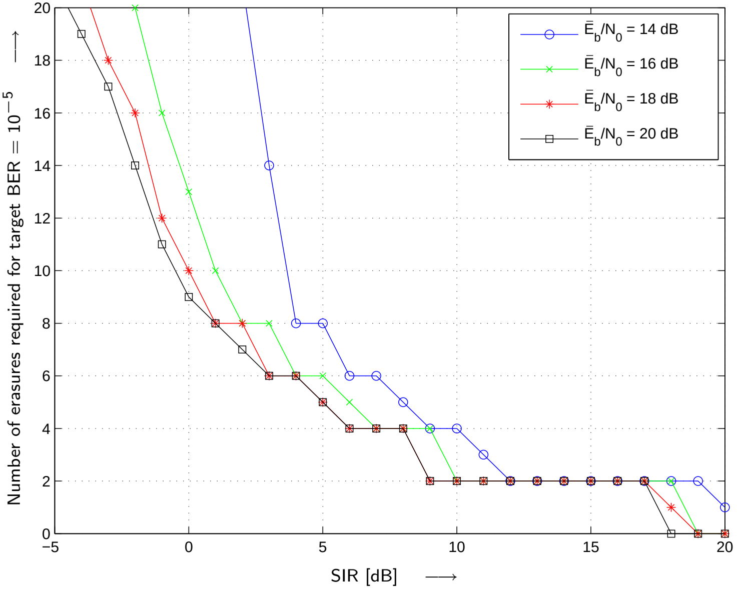
10% Outage BER. 100 CM1 channel realizations. No interference ($\mathbf{J} = \mathbf{0}$).

Results: BER with Tone Interference



Varying SIR, interferer position. $N_i = 1$, AWGN channel, $\bar{E}_b/\mathcal{N}_0 = 4.0$ dB, $R_c = 1/2$.

Results: Erasures



$N_i = 1$, tone position 52.5, $R_c = 1/2$. Average over 100 CM1 realizations.

Conclusions

Analysis

Developed methods to estimate

- Outage BER
- Average BER
- Effect of interference

These methods apply to any OFDM system in quasi-static channel

Results

- Can approximate MB-OFDM performance without simulations
- Tone interference may have a strong impact on MB-OFDM
- We can use erasure decoding to improve performance

More detailed journal preprint at www.ece.ubc.ca/∼csnow/